View Factors

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Introduction

This document provides a summary of the Octave/MATLAB routines that have been implemented for calculation of view factors for a variety of geometries.

These functions can be used to evaluate view factors for input to TNSolver using the Radiation Enclosure command block.
Summation Rule (Equation (13.4), page 830 in [BLID11]):

\[ \sum_{i=1}^{N} F_{ij} = 1 \]

Reciprocity Rule (Equation (13.3), page 829 in [BLID11]):

\[ A_i F_{ij} = A_j F_{ji} \]

Addition of View Factors for Subdivided Surfaces (Equation (13.5), page 833 and Figure 13.7, page 835 in [BLID11]):

\[ F_{i(j)} = \sum_{k=1}^{N} F_{ik} \]
View Factor for Coaxial Parallel Plates

\[ X = \frac{a}{c} \]

\[ Y = \frac{b}{c} \]

See Table 13.2, page 833 in [BLID11] or Table 10.3, page 546 in [LL12]
Also see the web site: A Catalog of Radiation Heat Transfer Configuration Factors, by John R. Howell, specifically C-11: Identical, parallel, directly opposed rectangles.
View Factor for Coaxial Parallel Plates (continued)

\[ F_{1-2} = \frac{2}{\pi XY} \left\{ \ln \left[ \frac{(1 + X^2) (1 + Y^2)}{1 + X^2 + Y^2} \right]^{1/2} \right. \\
+ X \sqrt{1 + Y^2} \tan^{-1} X \frac{X}{\sqrt{1 + Y^2}} \\
+ Y \sqrt{1 + X^2} \tan^{-1} Y \frac{Y}{\sqrt{1 + X^2}} \\
\left. - X \tan^{-1} X - Y \tan^{-1} Y \right\} \]
View Factor for Orthogonal Plate to Plate

\[ H = \frac{h}{l} \]

\[ W = \frac{w}{l} \]

See Table 13.2, page 833 in [BLID11] or Table 10.3, page 546 in [LL12]

Also see the web site: A Catalog of Radiation Heat Transfer Configuration Factors, by John R. Howell, specifically C-14: Two finite rectangles of same length, having one common edge, and at an angle of 90° to each other.
View Factor for Orthogonal Plate to Plate (continued)

\[ F_{1-2} = \frac{1}{\pi W} \left\{ W \tan^{-1} \frac{1}{W} + H \tan^{-1} \frac{1}{H} \right. \]

\[ - \sqrt{H^2 + W^2} \tan^{-1} \sqrt{\frac{1}{H^2 + W^2}} \]

\[ + \frac{1}{4} \ln \left( \frac{(1 + W^2) (1 + H^2)}{1 + W^2 + H^2} \right) \]

\[ \times \left[ \frac{W^2 (1 + W^2 + H^2)}{(1 + W^2) (W^2 + H^2)} \right]^{W^2} \left[ \frac{H^2 (1 + H^2 + W^2)}{(1 + H^2) (H^2 + W^2)} \right]^{H^2} \]
View Factor for Coaxial Parallel Disks

\[ R_i = \frac{r_i}{L}, \quad R_j = \frac{r_j}{L} \]

\[ S = 1 + \frac{1 + R_j^2}{R_i^2} \]

\[ F_{ij} = \frac{1}{2} \left\{ S - \sqrt{S^2 - 4 \left( \frac{r_j}{r_i} \right)^2} \right\} \]

See Table 13.2, page 833 in [BLID11] or Table 10.3, page 546 in [LL12].
Also see the web site: A Catalog of Radiation Heat Transfer Configuration Factors, by
John R. Howell, specifically C-41: Disk to parallel coaxial disk of unequal radius.
View Factor for Conical Frustum Wall to its Base

\[ H = \frac{h}{r_b} \quad R = \frac{r_b}{r_t} \]

\[ X = 1 + R^2 + H^2 \]

\[ F_{wb} = \frac{2R^2 - X + \sqrt{X^2 - 4R^2}}{2(1 + R) \sqrt{X - 2R}} \]

See the web site: A Catalog of Radiation Heat Transfer Configuration Factors, by John R. Howell, specifically C-112: Interior of frustum of right circular cone to base.
View Factor for Disk to Coaxial Cone

\[ \alpha = \tan^{-1} \frac{r_c}{h} \]

\[ S = \frac{r_d}{L} \quad R = \frac{r_c}{r_d} \quad X = S + R \cot \alpha \]

\[ A = \sqrt{X^2 + (1 + R)^2} \]

\[ B = \sqrt{X^2 + (1 - R)^2} \]

\[ C = \sqrt{\cos \alpha + S \sin \alpha} \]

\[ D = \sqrt{\cos \alpha - S \sin \alpha} \]

\[ E = R \cot \alpha - S \]

See the web site: A Catalog of Radiation Heat Transfer Configuration Factors, by John R. Howell, specifically C-48: Disk to coaxial cone.
View Factor for Disk to Coaxial Cone (continued)

For \( \alpha \geq \tan^{-1}\frac{1}{S} \):

\[
F_{dc} = \frac{1}{2} \left\{ R^2 + X^2 + 1 - \sqrt{(1 + R^2 + X^2)^2 - 4R^2} \right\}
\]

For \( \alpha < \tan^{-1}\frac{1}{S} \):

\[
F_{dc} = \frac{1}{\pi} \left\{ -AB \tan^{-1} \frac{AC}{BD} + \left( 1 + S^2 \right) \tan^{-1} \frac{C}{D} \right. \\
+ \frac{\sin \alpha}{\cos^2 \alpha} \left[ X \tan^{-1} \frac{CD}{X} + S^2 \tan^{-1} \frac{CD}{S} + (CD)^2 \left( \tan^{-1} \frac{X}{CD} - \tan^{-1} \frac{S}{CD} \right) \right] \\
+ \left[ \frac{R(X + S)}{\sin 2\alpha} - SR \tan \alpha \right] \cos^{-1} \left( -S \tan \alpha \right) \right\}
\]
View Factor for Plate to Cylinder

\[
X = \frac{H}{r} \quad Y = \frac{W}{r} \quad Z = \frac{L}{r}
\]

\[
A = Z^2 + X^2 + \zeta^2 - 1
\]

\[
B = Z^2 - X^2 - \zeta^2 + 1
\]

\[
F_{\text{plate-cyl}} = \frac{2}{Y} \int_0^{Y/2} f(\zeta) d\zeta
\]

See [SC78] and the web site: A Catalog of Radiation Heat Transfer Configuration Factors, by John R. Howell, specifically C-74: Finite-length cylinder to rectangle with two edges parallel to cylinder axis and of length equal to cylinder.
View Factor for Plate to Cylinder (continued)

\[ f(\xi) = \frac{X}{X^2 + \xi^2} - \frac{X}{\pi (X^2 + \xi^2)} \]
\[
\times \left\{ \cos^{-1} \frac{B}{A} - \frac{1}{2Z} \left[ \sqrt{A^2 + 4Z^2} \cos^{-1} \left( \frac{B}{A\sqrt{X^2 + \xi^2}} \right) \right. \right.
\]
\[
\left. + B\sin^{-1} \left( \frac{1}{\sqrt{X^2 + \xi^2}} - \frac{\pi A}{2} \right) \right\} \]
References I

Introduction to Heat Transfer.

Available at: http://ahtt.mit.edu.

Radiation Heat Transfer.