View Factors

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Introduction

This document provides a summary of the Octave/MATLAB routines that have been implemented for calculation of view factors for a variety of geometries.

These functions can be used to evaluate view factors for input to TNSolver using the `Radiation Enclosure` command block.

View Factor Properties

Summation Rule (Equation (13.4), page 830 in [BLID11]):

$$\sum_{i=1}^{N} F_{ij} = 1$$

Reciprocity Rule (Equation (13.3), page 829 in [BLID11]):

$$A_i F_{ij} = A_j F_{ji}$$

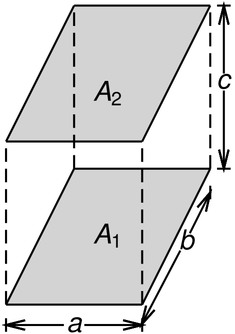
Addition of View Factors for Subdivided Surfaces (Equation (13.5), page 833 and Figure 13.7, page 835 in [BLID11]):

$$F_{i(j)} = \sum_{k=1}^{N} F_{ik}$$

View Factor for Coaxial Parallel Plates

$$X = \frac{a}{c}$$

$$Y = \frac{b}{c}$$



See Table 13.2, page 833 in [BLID11] or Table 10.3, page 546 in [LL12]
Also see the web site: A Catalog of Radiation Heat Transfer Configuration Factors, by John R. Howell, specifically C-11: Identical, parallel, directly opposed rectangles.

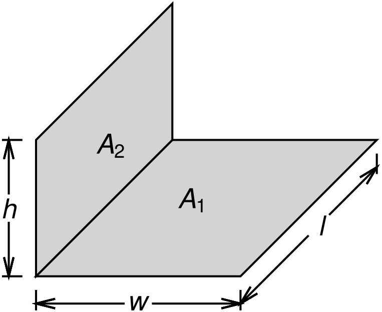
View Factor for Coaxial Parallel Plates (continued)

$$
\begin{aligned}
F_{1-2} = \frac{2}{\pi XY} \Bigg\{ & \ln \left[\frac{\left(1 + X^2\right)\left(1 + Y^2\right)}{1 + X^2 + Y^2} \right]^{1/2} \\
& + X\sqrt{1 + Y^2}\tan^{-1}\frac{X}{\sqrt{1 + Y^2}} \\
& + Y\sqrt{1 + X^2}\tan^{-1}\frac{Y}{\sqrt{1 + X^2}} \\
& - X\tan^{-1} X - Y\tan^{-1} Y \Bigg\}
\end{aligned}
$$

View Factor for Orthogonal Plate to Plate

$$H = \frac{h}{l}$$

$$W = \frac{w}{l}$$



See Table 13.2, page 833 in [BLID11] or Table 10.3, page 546 in [LL12]
Also see the web site: A Catalog of Radiation Heat Transfer Configuration Factors, by John R. Howell, specifically C-14: Two finite rectangles of same length, having one common edge, and at an angle of 90° to each other.

View Factor for Orthogonal Plate to Plate (continued)

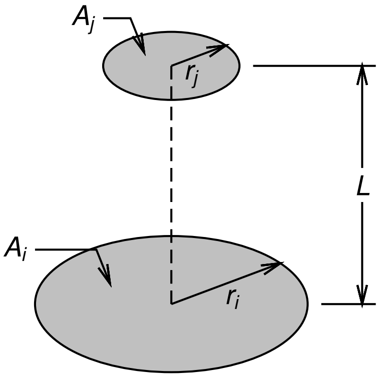
$$F_{1-2} = \frac{1}{\pi W} \left\{ W \tan^{-1} \frac{1}{W} + H \tan^{-1} \frac{1}{H} \right.$$
$$- \sqrt{H^2 + W^2} \tan^{-1} \sqrt{\frac{1}{H^2 + W^2}}$$
$$+ \frac{1}{4} \ln \left(\frac{\left(1 + W^2\right)\left(1 + H^2\right)}{1 + W^2 + H^2} \right.$$
$$\left. \left. \times \left[\frac{W^2\left(1 + W^2 + H^2\right)}{\left(1 + W^2\right)\left(W^2 + H^2\right)} \right]^{W^2} \left[\frac{H^2\left(1 + H^2 + W^2\right)}{\left(1 + H^2\right)\left(H^2 + W^2\right)} \right]^{H^2} \right) \right\}$$

View Factor for Coaxial Parallel Disks

$$R_i = \frac{r_i}{L}, \; R_j = \frac{r_j}{L}$$

$$S = 1 + \frac{1 + R_j^2}{R_i^2}$$

$$F_{ij} = \frac{1}{2}\left\{ S - \sqrt{S^2 - 4\left(\frac{r_j}{r_i}\right)^2} \right\}$$



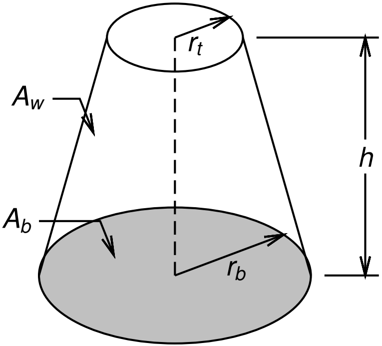
See Table 13.2, page 833 in [BLID11] or Table 10.3, page 546 in [LL12]
Also see the web site: A Catalog of Radiation Heat Transfer Configuration Factors, by John R. Howell, specifically C-41: Disk to parallel coaxial disk of unequal radius.

View Factor for Conical Frustum Wall to its Base

$$H = \frac{h}{r_b} \quad R = \frac{r_b}{r_t}$$

$$X = 1 + R^2 + H^2$$

$$F_{wb} = \frac{2R^2 - X + \sqrt{X^2 - 4R^2}}{2\,(1 + R)\,\sqrt{X - 2R}}$$



See the web site: A Catalog of Radiation Heat Transfer Configuration Factors, by John R. Howell, specifically C-112: Interior of frustum of right circular cone to base.

View Factor for Disk to Coaxial Cone

$$\alpha = \tan^{-1}\frac{r_c}{h}$$

$$S = \frac{r_d}{L} \quad R = \frac{r_c}{r_d} \quad X = S + R\cot\alpha$$

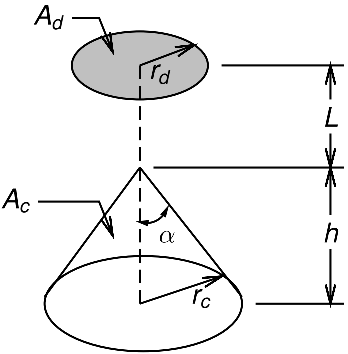
$$A = \sqrt{X^2 + (1+R)^2}$$

$$B = \sqrt{X^2 + (1-R)^2}$$

$$C = \sqrt{\cos\alpha + S\sin\alpha}$$

$$D = \sqrt{\cos\alpha - S\sin\alpha}$$

$$E = R\cot\alpha - S$$



See the web site: A Catalog of Radiation Heat Transfer Configuration Factors, by John R. Howell, specifically C-48: Disk to coaxial cone.

View Factor for Disk to Coaxial Cone (continued)

For $\alpha \geq \tan^{-1}\frac{1}{S}$:

$$F_{dc} = \frac{1}{2}\left\{R^2 + X^2 + 1 - \sqrt{\left(1 + R^2 + X^2\right)^2 - 4R^2}\right\}$$

For $\alpha < \tan^{-1}\frac{1}{S}$:

$$F_{dc} = \frac{1}{\pi}\left\{-AB\tan^{-1}\frac{AC}{BD} + \left(1 + S^2\right)\tan^{-1}\frac{C}{D}\right.$$
$$+\frac{\sin\alpha}{\cos^2\alpha}\left[XE\tan^{-1}\frac{CD}{X} + S^2\tan^{-1}\frac{CD}{S} + (CD)^2\left(\tan^{-1}\frac{X}{CD} - \tan^{-1}\frac{S}{CD}\right)\right]$$
$$\left. + \left[\frac{R\,(X+S)}{\sin 2\alpha} - SR\tan\alpha\right]\cos^{-1}\left(-S\tan\alpha\right)\right\}$$

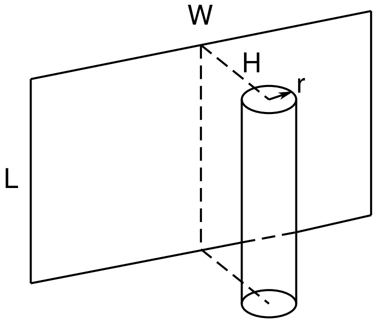
View Factor for Plate to Cylinder

$$X = \frac{H}{r} \quad Y = \frac{W}{r} \quad Z = \frac{L}{r}$$

$$A = Z^2 + X^2 + \zeta^2 - 1$$

$$B = Z^2 - X^2 - \zeta^2 + 1$$

$$F_{\text{plate-cyl}} = \frac{2}{Y} \int_0^{Y/2} f(\zeta) d\zeta$$



See [SC78] and the web site: A Catalog of Radiation Heat Transfer Configuration Factors, by John R. Howell, specifically C-74: Finite-length cylinder to rectangle with two edges parallel to cylinder axis and of length equal to cylinder.

View Factor for Plate to Cylinder (continued)

$$f(\xi) = \frac{X}{X^2+\xi^2} - \frac{X}{\pi\left(X^2+\xi^2\right)} \times \left\{ \cos^{-1}\frac{B}{A} - \frac{1}{2Z}\left[\sqrt{A^2+4Z^2}\cos^{-1}\left(\frac{B}{A\sqrt{X^2+\xi^2}}\right) + B\sin^{-1}\left(\frac{1}{\sqrt{X^2+\xi^2}}\right) - \frac{\pi A}{2}\right]\right\}$$

References I

[BLID11] T.L. Bergman, A.S. Lavine, F.P. Incropera, and D.P. DeWitt.
Introduction to Heat Transfer.
John Wiley & Sons, New York, sixth edition, 2011.

[LL12] J. H. Lienhard, IV and J. H. Lienhard, V.
A Heat Transfer Textbook.
Phlogiston Press, Cambridge, Massachusetts, fourth edition, 2012.
Available at: `http://ahtt.mit.edu`.

[SC78] E. M. Sparrow and R. D. Cess.
Radiation Heat Transfer.
McGraw-Hill, New York, augmented edition, 1978.