

Radiation Heat Transfer Experiment

Thermal Network Solution with TNSolver

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Outline

- ▶ Math Model
- ▶ Geometry
- ▶ Test Data
- ▶ Thermal Network Model Solution with TNSolver
- ▶ An Example Calculation

Convection Correlations

Math Model

The heat flow rate is:

$$Q = hA(T_s - T_\infty)$$

where h is the heat transfer coefficient, T_s is the surface temperature and T_∞ is the fluid temperature.

Correlations in terms of the Nusselt number are often used to determine h :

$$Nu = \frac{hL_c}{k} \qquad h = \frac{kNu}{L_c}$$

where L_c is a characteristic length associated with the fluid flow geometry.

External Natural Convection: Horizontal Plate Up

Math Model

The heat transfer coefficient for laminar flow, $10^4 \lesssim Ra_L \lesssim 10^7$, from a hot plate, $T_s > T_\infty$, is (see Equation (9.30), p. 578, in [BLID11]):

$$\overline{Nu}_L = 0.54 Ra_L^{1/4} \quad (1)$$

and for turbulent flow, $10^7 \lesssim Ra_L \lesssim 10^{11}$, from a hot plate is (see Equation (9.31), p. 578, in [BLID11]):

$$\overline{Nu}_L = 0.15 Ra_L^{1/3} \quad (2)$$

External Natural Convection: Horizontal Plate Up

Math Model

For a cold plate, $T_s < T_\infty$, $10^4 \lesssim Ra_L \lesssim 10^9$, the correlation is (see Equation (9.32), p. 578, in [BLID11]):

$$\overline{Nu}_L = 0.52 Ra_L^{1/5} \quad (3)$$

Then the Rayleigh number, Ra_L , where $L = A/P$, is:

$$Ra_L = GrPr = \frac{g\rho^2 c\beta L^3 (T_s - T_\infty)}{k\mu} = \frac{g\beta L^3 (T_s - T_\infty)}{\nu\alpha} \quad (4)$$

Note that the fluid properties are evaluated at the film temperature, T_f :

$$T_f = \frac{T_s + T_\infty}{2} \quad (5)$$

External Natural Convection: Horizontal Plate Down

Math Model

The heat transfer coefficient for laminar flow, $10^4 \lesssim Ra_L \lesssim 10^7$, from a cold plate, $T_s < T_\infty$, is (see Equation (9.30), p. 578, in [BLID11]):

$$\overline{Nu}_L = 0.54 Ra_L^{1/4} \quad (6)$$

and for turbulent flow, $10^7 \lesssim Ra_L \lesssim 10^{11}$, from a cold plate is (see Equation (9.31), p. 578, in [BLID11]):

$$\overline{Nu}_L = 0.15 Ra_L^{1/3} \quad (7)$$

External Natural Convection: Horizontal Plate Down

Math Model

For a hot plate, $T_s > T_\infty$, $10^4 \lesssim Ra_L \lesssim 10^9$, the correlation is (see Equation (9.32), p. 578, in [BLID11]):

$$\overline{Nu}_L = 0.52 Ra_L^{1/5} \quad (8)$$

Then the Rayleigh number, Ra_L , where $L = A/P$, is:

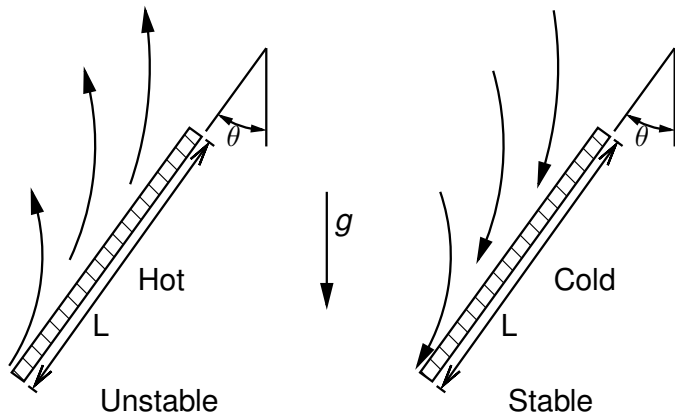
$$Ra_L = GrPr = \frac{g\rho^2 c\beta L^3 (T_s - T_\infty)}{k\mu} = \frac{g\beta L^3 (T_s - T_\infty)}{\nu\alpha} \quad (9)$$

Note that the fluid properties are evaluated at the film temperature, T_f :

$$T_f = \frac{T_s + T_\infty}{2} \quad (10)$$

External Natural Convection: Inclined Plate Up

Math Model



External Natural Convection: Inclined Plate Up

Math Model

Stable Case, $T_s < T_\infty$:

The Nusselt number correlation for natural convection flow from a vertical plate (see Equation (9.26) and Equation (9.27), p. 573 in [BLID11] or [CC75]), with Rayleigh number:

$$Ra_L = GrPr = \frac{(g \cos \theta) \rho^2 c \beta L^3 (T_s - T_\infty)}{k \mu}$$

Note that the fluid properties are evaluated at the film temperature, T_f :

$$T_f = \frac{T_s + T_\infty}{2}$$

External Natural Convection: Inclined Plate Up

Math Model

Unstable Case, $T_s > T_\infty$:

The approach of Raithby and Hollands [RH98] is used. In this approach the heat transfer coefficient is evaluated for both a vertical plate with $g \cos \theta$ (see the stable case) and a horizontal plate with $g \cos(90 - \theta)$. The maximum of the two is then used.

Enclosure Radiation: The Exchange Factor, \mathcal{F}

Math Model

The exchange factor concept is based on proposing that there is a parameter, \mathcal{F}_{ij} , based on surface properties and enclosure geometry, that determines the radiative heat exchange between two surfaces [Hot54, HS67] :

$$Q_{ij} = A_i \mathcal{F}_{ij} \sigma (T_i^4 - T_j^4)$$

\mathcal{F} is known by many names in the literature: script-F, gray body configuration factor, transfer factor and Hottel called it the over-all interchange factor.

For an enclosure with N surfaces, the net heat flow rate, Q_i , for surface i , is:

$$Q_i = \sum_{j=1}^N A_i \mathcal{F}_{ij} (\sigma T_i^4 - \sigma T_j^4) = \sum_{j=1}^N A_i \mathcal{F}_{ij} (E_{bi} - E_{bj})$$

Exchange Factor, \mathcal{F} , Properties

Math Model

Reciprocity:

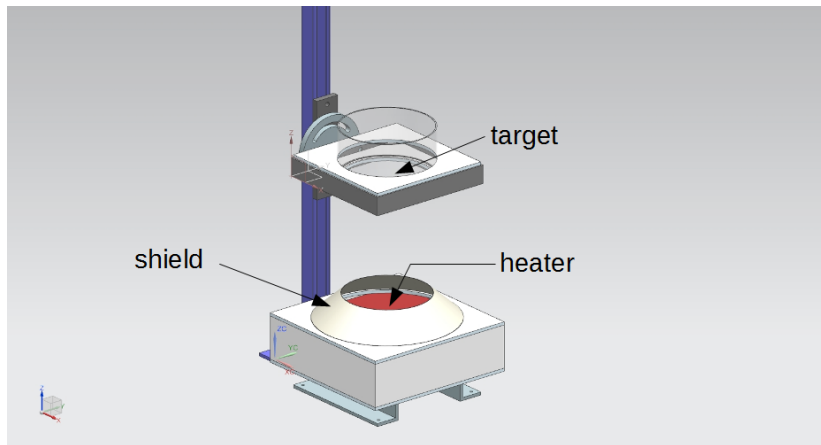
$$A_i \mathcal{F}_{ij} = A_j \mathcal{F}_{ji}$$

Summation:

$$\sum_{j=1}^N \mathcal{F}_{ij} = \epsilon_i$$

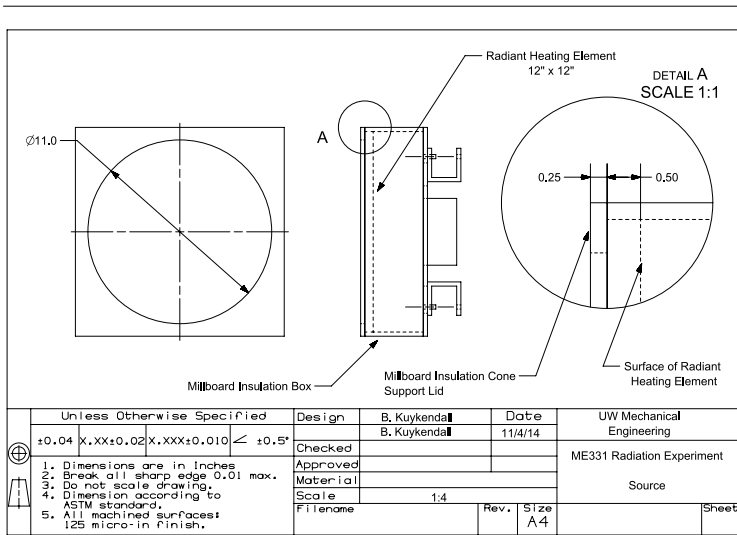
Radiation Heat Transfer Experiment

Geometry

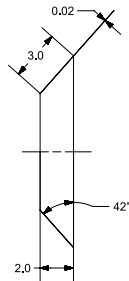
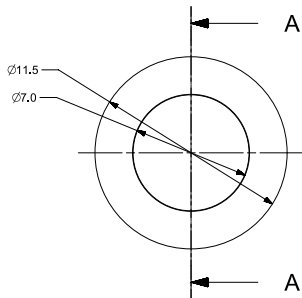


Heater

Geometry



Shield Geometry

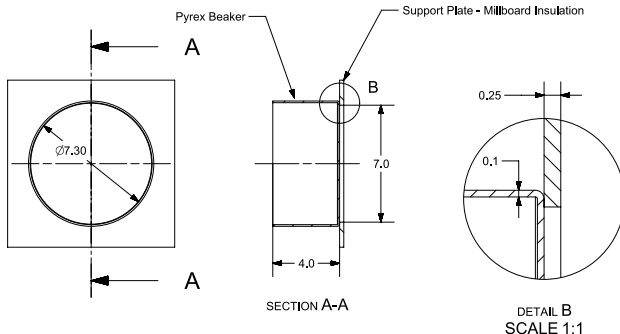




SECTION A-A

	± 0.04 X.XX ± 0.02 X.XXX ± 0.010 $\leq \pm 0.5^\circ$			Design	B. Kuykendall	Date	11/4/14	UW Mechanical Engineering
				Drawn	B. Kuykendall			
				Checked				
				Material	Galvanized Steel		Cone	
				Scale	1:4			
			Filename		Rev.	Size A4	Sheet	

1. Dimensions are in Inches
2. Break all sharp edge 0.01 max.
3. Do not scale drawing.
4. Dimension according to ASTM standard.
5. All machined surfaces: 125 micro-in finish.

Target Geometry



 	Unless Otherwise Specified			Design	B. Kuykendall	Date	UW Mechanical Engineering	
	± 0.04	$X.XX \pm 0.02$	$X.XXX \pm 0.010$	$\leq \pm 0.5^\circ$	Drawn	B. Kuykendall	11/4/14	ME331 Radiation Experiment
	1. Dimensions are in Inches				Approved			
	2. Break all sharp edge 0.01 max.				Material	Pyrex / Millboard		
	3. Do not scale drawing.				Scale	1:4		
4. Dimension according to ASTM standard.				Filename		Rev.	Size	Sheet
5. All machined surfaces: 125 micro-in finish.							A4	

Measurements

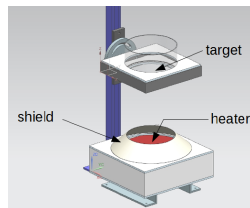
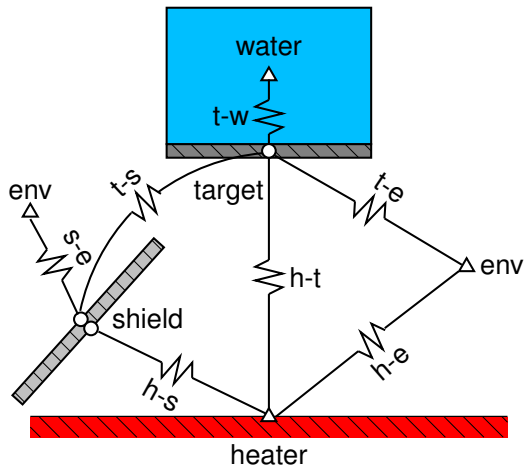
Test Data

For each target material:

- ▶ Heater temperature
- ▶ Water evaporation rate
- ▶ Ambient air temperature
- ▶ Distance from target to heater

TNSolver Thermal Network Model

TNSolver Input File



External Natural Convection (ENC): Horizontal Plate Up

TNSolver Input File

$$Q_{ij} = hA(T_s - T_\infty)$$

Heat transfer coefficient, h , is evaluated using the correlation for external natural convection from horizontal plate facing up.

```
Begin Conductors
!           Ts  Tinf
! label    type  nd_i nd_j parameters
      (S) ENChplateup (S) (S) (S) (R) (R) ! mat, L=A/P, A
End Conductors
```

Note that Ra, Nu and h are reported in the output file.

External Natural Convection (ENC): Horizontal Plate Down

TNSolver Input File

$$Q_{ij} = hA(T_s - T_\infty)$$

Heat transfer coefficient, h , is evaluated using the correlation for external natural convection from horizontal plate facing down.

```
Begin Conductors
!
!           Ts  Tinf
! label    type    nd_i nd_j parameters
! (S) ENChplatedown (S) (S) (S) (R) (R) ! mat, L=A/P, A
End Conductors
```

Note that Ra, Nu and h are reported in the output file.

External Natural Convection (ENC): Inclined Plate Up

TNSolver Input File

$$Q_{ij} = hA(T_s - T_\infty)$$

Heat transfer coefficient, h , is evaluated using the correlation for external natural convection from horizontal plate facing down.

```
Begin Conductors
!           Ts  Tinf
! label    type  nd_i nd_j parameters
  (S) ENCiplateup (S)  (S) (S) (R) (R) (R) ! mat, L, angle, A

End Conductors
```

Note that Ra, Nu and h are reported in the output file.

Radiation Enclosure

TNSolver Input File

Each radiation enclosure is described by the surface labels, emissivities, areas and the view factor matrix $[F]$:

```
Begin Radiation Enclosure

! label  emiss  area  view factor matrix entries
  (S)    (R)    (R)    (R ...)
```



```
End Radiation Enclosure
```

The generated radiation conductors are reported in the output file.

Example Input File

TNSolver Input File

Begin Solution Parameters

```
title = Radiation Heat Transfer Experiment - Black Target
type = steady
nonlinear convergence = 1.0e-8
maximum nonlinear iterations = 50
```

End Solution Parameters

Begin Conductors

```
! Conduction through the beaker wall, 0.1" thick pyrex glass
  t-bbin conduction targ bbin 0.14 0.00254 0.024829 ! k L A
! Convection from beaker to water
! label type      nd_i  nd_j  mat  L  A
t-w ENChplateup  bbin  water  water 0.04445 0.02483
! Convection from target to air
! label type      nd_i  nd_j  mat  L  A
t-air ENChplatedown targ env  air 0.0889 0.0248
! Convection from outer shield to air
! label type      nd_i  nd_j  mat  L  theta  A
s-air ENCiplateup s_out env  air 0.0508 48.0 0.056439
! Conduction from inner to outer side of shield
shield conduction s_in s_out steel 0.001 0.056439
```

End Conductors

Example Input File (continued)

TNSolver Input File

Begin Radiation Enclosure

```
! surf emiss  A      Fij
htr  0.92 0.06701 0.0      0.1264  0.68415 0.0      0.1893
targ 0.95 0.02482 0.34132 0.0      0.00603 0.1031 0.5494
s_in  0.28 0.05643 0.81231 0.00265 0.12278 0.0      0.0622
s_out 0.28 0.05643 0.0      0.0453  0.0      0.0      0.9546
env   1.0  0.1184  0.10711 0.1151  0.02965 0.4547 0.2933
```

End Radiation Enclosure

Begin Boundary Conditions

```
! type      Tb      Node(s)
fixed_T     23.0     env
fixed_T     88.3     water
fixed_T     515.0    htr
```

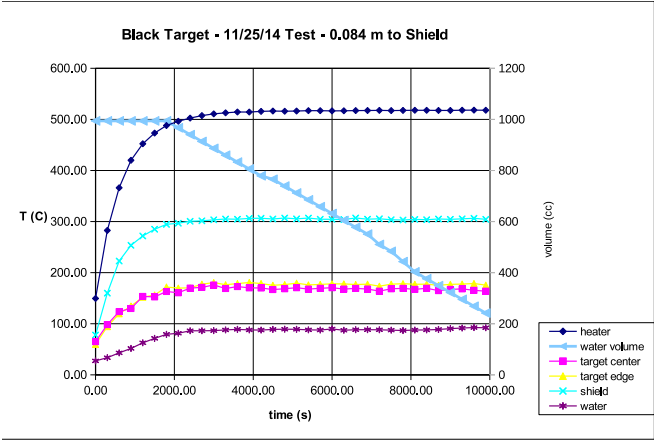
End Boundary Conditions

Black Target Test Data

Results

Heater Emissivity = 0.92 @ 500 C

Target Emissivity = 0.95 (black) @ 150 C



Black Target Water Evaporation Rate

Results

MATLAB: polyfit(time, vol, 1)

Evaporation Rate = $0.093793\text{E-}6 \text{ m}^3/\text{s}$

Water density (at 88.3 C) $\rho = 967 \text{ kg}/\text{m}^3$

Water latent heat of vaporization, $h_{fg} = 2,256,000 \text{ J}/\text{kg}$

$Q = 967 * 0.093793\text{E-}6 * 2,256,000 = 205 \text{ W}$

Black Target View Factors, F_{ij}

Results

	Area (m^2)	Emissivity, ϵ
heater	0.06701	0.92
target	0.02483	0.95
shield	0.05644	0.28
env	0.1185	1.0

View Factor Matrix, $[F]$

	heater	target	s_in	s_out	env
heater	0	0.1265	0.6842	0	0.1894
target	0.3413	0	0.006039	0.1032	0.5494
s_in	0.8123	0.002657	0.1228	0	0.06224
s_out	0	0.0454	0	0	0.9546
env	0.1071	0.1151	0.02965	0.4547	0.2934

Conclusion

- ▶ Math model components for heat transfer analysis
- ▶ TNSolver input file specifics
- ▶ Demonstrated simulation results for an experimental data set

Questions?

Surface Properties for Radiation Heat Transfer

Appendix

- ▶ All surfaces emit thermal radiation.
- ▶ The emittance, ϵ , is the ratio of actual energy emitted to that of a black surface at the same temperature.
- ▶ All radiation impinging on a surface will either be reflected, absorbed or transmitted.
- ▶ Reflectance or reflectivity, ρ , is the amount reflected.
- ▶ Absorptance or absorptivity, α , is the amount absorbed.
- ▶ Transmittance or transmissivity, τ , is the amount transmitted through the material.

Surface Properties for Radiation Heat Transfer (continued)

Appendix

Summation property for all incident radiation:

$$\rho + \alpha + \tau = 1$$

An **opaque** surface has $\tau = 0$, so $\rho + \alpha = 0$.

Kirchoff's law provides that $\epsilon = \alpha$ for gray, diffuse surfaces.

Diffuse is a modifier which means the property is not a function of direction.

Gray is a modifier indicating no dependence on wavelength.

Spectral is a modifier which means dependence on wavelength.

Specular is a modifier which means mirror-like reflection.

Characteristics of Real Surfaces

Appendix

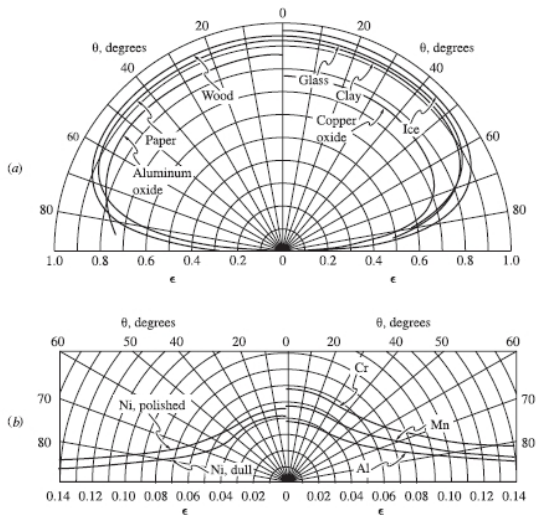


Figure 8.4 Directional variation of surface emittances: (a) for several nonmetals; (b) for several metals. (From Schmidt and Eckert, 1935.)

View Factor Properties

Appendix

Summation Rule (Equation (13.4), page 830 in [BLID11]):

$$\sum_{i=1}^N F_{ij} = 1$$

Reciprocity Rule (Equation (13.3), page 829 in [BLID11]):

$$A_i F_{ij} = A_j F_{ji}$$

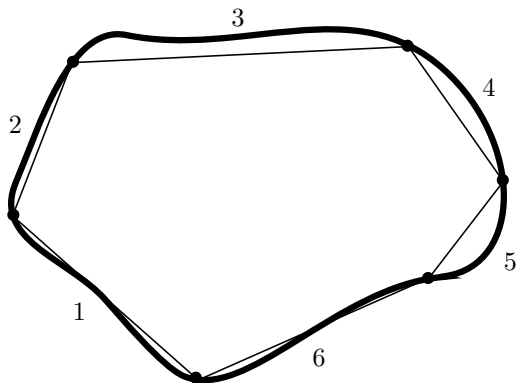
Addition of View Factors for Subdivided Surfaces (Equation (13.5), page 833 and Figure 13.7, page 835 in [BLID11]):

$$F_{i(j)} = \sum_{k=1}^N F_{ik}$$

Net Radiation Method for Enclosures

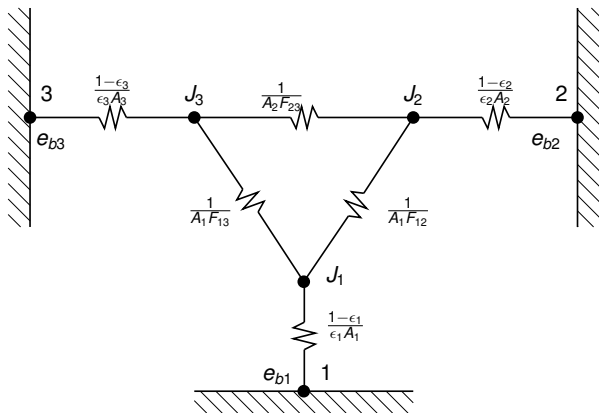
Appendix

- ▶ The enclosure geometry is approximated by a set of N ideal surfaces
 - ▶ Diffuse gray emission/absorption
 - ▶ Diffuse gray reflections
 - ▶ Each surface is isothermal, with uniform heat flux



Oppenheim Network for Three Surfaces

Appendix



A.K. Oppenheim, "Radiation Analysis by the Network Method,"
Transactions of the ASME, vol. 78, pp. 725–735, 1956

The Surface Radiosity

Appendix

Radiosity, J_i , for surface i is:

$$J_i = \underbrace{\epsilon_i \sigma T_i^4}_{\text{emmission}} + \underbrace{\rho_i G_i}_{\text{reflected irradiation}}$$

Surface irradiation, G_i is:

$$G_i = \sum_{j=1}^N F_{ij} J_j$$

Then noting that $\rho_i = 1 - \epsilon_i$, the equation for the radiosity from surface i is:

$$J_i - (1 - \epsilon_i) \left(\sum_{j=1}^N F_{ij} J_j \right) = \epsilon_i \sigma T_i^4$$

System of Equations for the Radiosity

Appendix

Rearrangement of the radiosity equation for surface i leads to:

$$\sum_{i=1}^N [\delta_{ij} - (1 - \epsilon_i) F_{ij}] J_j = \epsilon_i \sigma T_i^4$$

Combining the radiosity equations for each surface in the enclosure leads to a system of equations:

$$[A] \{J\} = \{b\}$$

Where:

$$A_{ij} = \delta_{ij} - (1 - \epsilon_i) F_{ij}$$

$$b_i = \epsilon_i \sigma T_i^4$$

The surface temperature, T_i , is provided by the solution of the energy conservation equation.

Application of Radiosity to Heat Conduction Equation

Appendix

Once the radiosity, J_i is known for a surface, the irradiation is evaluated:

$$G_i = \sum_{j=1}^N F_{ij} J_j$$

This is then used to evaluate the net radiant thermal energy for surface i :

$$q_i|_{radiation} = \epsilon_i \sigma T_i^4 - \alpha_i G_i$$

This heat flux boundary condition is then applied to the energy conservation equation solution so that updated surface temperatures can be determined.

Iteration between the energy equation and the radiosity is continued until convergence.

Matrix Form of the Radiosity Equation

Appendix

$$\sum_{i=1}^N [\delta_{ij} - (1 - \epsilon_i) F_{ij}] J_j = \epsilon_i \sigma T_i^4$$
$$([I] - [\rho])[F] \{J\} = [\epsilon] \{E_b\}$$

where, for $N = 3$:

$$\{J\} = \begin{Bmatrix} J_1 \\ J_2 \\ J_3 \end{Bmatrix} \quad \{\sigma T^4\} = \begin{Bmatrix} \sigma T_1^4 \\ \sigma T_2^4 \\ \sigma T_3^4 \end{Bmatrix} = \{E_b\} = \begin{Bmatrix} E_{b1} \\ E_{b2} \\ E_{b3} \end{Bmatrix}$$

$$[\epsilon] = \begin{bmatrix} \epsilon_1 & 0 & 0 \\ 0 & \epsilon_2 & 0 \\ 0 & 0 & \epsilon_3 \end{bmatrix} \quad [F] = \begin{bmatrix} F_{11} & F_{12} & F_{13} \\ F_{21} & F_{22} & F_{23} \\ F_{31} & F_{32} & F_{33} \end{bmatrix} \quad [I] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$[\rho] = \begin{bmatrix} \rho_1 & 0 & 0 \\ 0 & \rho_2 & 0 \\ 0 & 0 & \rho_3 \end{bmatrix} = [I] - [\epsilon] = \begin{bmatrix} 1 - \epsilon_1 & 0 & 0 \\ 0 & 1 - \epsilon_2 & 0 \\ 0 & 0 & 1 - \epsilon_3 \end{bmatrix}$$

Enclosure Heat Flow from Radiosity

Appendix

$$([I] - [\rho][F])\{J\} = [\epsilon]\{E_b\}$$

Using the definition of radiosity:

$$\{J\} = [\epsilon]\{E_b\} + [\rho]\{G\} = [\epsilon]\{E_b\} + [\rho][F]\{J\}$$

and the net surface heat flux ($\alpha = \epsilon$):

$$\{q\} = [\epsilon]\{E_b\} - [\alpha]\{G\} = [\epsilon]\{E_b\} - [\epsilon][F]\{J\}$$

then some algebraic manipulation leads to:

$$([I] - [F][\rho])[\epsilon]^{-1} \{q\} = ([I] - [F])\{E_b\}$$

or,

$$\{q\} = [\epsilon]([I] - [F][\rho])^{-1}([I] - [F])\{E_b\}$$

Exchange Factor, \mathcal{F}

Appendix

The exchange factor concept is based on proposing that there is a parameter, \mathcal{F}_{ij} , based on surface properties and enclosure geometry, that determines the radiative heat exchange between two surfaces [Hot54, HS67] :

$$Q_{ij} = A_i \mathcal{F}_{ij} \sigma (T_i^4 - T_j^4)$$

\mathcal{F} is known by many names in the literature: script-F, gray body configuration factor, transfer factor and Hottel called it the over-all interchange factor.

For an enclosure with N surfaces, the net heat flow rate, Q_i , for surface i , is:

$$Q_i = \sum_{j=1}^N A_i \mathcal{F}_{ij} (\sigma T_i^4 - \sigma T_j^4) = \sum_{j=1}^N A_i \mathcal{F}_{ij} (E_{bi} - E_{bj})$$

Exchange Factor, \mathcal{F} , Properties

Appendix

Reciprocity:

$$A_i \mathcal{F}_{ij} = A_j \mathcal{F}_{ji}$$

Summation:

$$\sum_{j=1}^N \mathcal{F}_{ij} = \epsilon_i$$

Matrix Form of Enclosure Radiation with \mathcal{F}

Appendix

$$Q_i = \sum_{j=1}^N A_i \mathcal{F}_{ij} (E_{bi} - E_{bj})$$

$$\{Q\} = [A] ([\epsilon] - [\mathcal{F}]) \{E_b\}$$

$$\{q\} = [A]^{-1} \{Q\} = ([\epsilon] - [\mathcal{F}]) \{E_b\}$$

where, for $N = 3$:

$$\{Q\} = \begin{Bmatrix} Q_1 \\ Q_2 \\ Q_3 \end{Bmatrix} \quad [A] = \begin{bmatrix} A_1 & 0 & 0 \\ 0 & A_2 & 0 \\ 0 & 0 & A_3 \end{bmatrix}$$

$$[\mathcal{F}] = \begin{bmatrix} \mathcal{F}_{11} & \mathcal{F}_{12} & \mathcal{F}_{13} \\ \mathcal{F}_{21} & \mathcal{F}_{22} & \mathcal{F}_{23} \\ \mathcal{F}_{31} & \mathcal{F}_{32} & \mathcal{F}_{33} \end{bmatrix} \quad \{E_b\} = \begin{Bmatrix} \sigma T_1^4 \\ \sigma T_2^4 \\ \sigma T_3^4 \end{Bmatrix} \quad [\epsilon] = \begin{bmatrix} \epsilon_1 & 0 & 0 \\ 0 & \epsilon_2 & 0 \\ 0 & 0 & \epsilon_3 \end{bmatrix}$$

Exchange Factors, \mathcal{F} , from View Factors, F

Appendix

The net heat flux using view factors, F_{ij} , is:

$$\{q\} = [\epsilon]([I] - [F][\rho])^{-1}([I] - [F])\{E_b\}$$

The net heat flux using exchange factors, \mathcal{F}_{ij} , is:

$$\{q\} = ([\epsilon] - [\mathcal{F}])\{E_b\}$$

The two enclosure heat fluxes are equal, so equating gives:

$$([\epsilon] - [\mathcal{F}])\{E_b\} = [\epsilon]([I] - [F][\rho])^{-1}([I] - [F])\{E_b\}$$

$$([\epsilon] - [\mathcal{F}]) = [\epsilon]([I] - [F][\rho])^{-1}([I] - [F])$$

$$[\mathcal{F}] = [\epsilon] \left([I] - ([I] - [F][\rho])^{-1}([I] - [F]) \right)$$

See [IB63] for an early reference to this method.

Linearization of Radiation Conductors

Appendix

The temperature is linearized using a two term Taylor series expansion about the previous iteration temperature, T^* :

$$T_i^4 \approx (T_i^*)^4 + (T_i - T_i^*) 4(T_i^*)^3$$

$$T_i^4 \approx 4(T_i^*)^3 T_i - 3(T_i^*)^4$$

$$T_j^4 \approx (T_j^*)^4 + (T_j - T_j^*) 4(T_j^*)^3$$

$$T_j^4 \approx 4(T_j^*)^3 T_j - 3(T_j^*)^4$$

The linearized form of the heat transfer rate is:

$$Q_{ij} = \sigma \mathcal{F}_{ij} A_i \left[4(T_i^*)^3 T_i - 3(T_i^*)^4 - 4(T_j^*)^3 T_j + 3(T_j^*)^4 \right]$$

$$Q_{ij} = \sigma \mathcal{F}_{ij} A_i \left\{ 4(T_i^*)^3 T_i - 4(T_j^*)^3 T_j \right\} - \sigma \mathcal{F}_{ij} A_i \left\{ 3(T_i^*)^4 - 3(T_j^*)^4 \right\}$$

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