Radiation Heat Transfer Experiment

Thermal Network Solution with TNSolver

Bob Cochran
Applied Computational Heat Transfer
Seattle, WA
rjc@heattransfer.org

ME 331 Introduction to Heat Transfer
University of Washington
December 1, 2015

Outline

- Math Model
- Geometry
- Test Data
- Thermal Network Model Solution with TNSolver
- An Example Calculation

Convection Correlations

Math Model

The heat flow rate is:

$$Q = hA(T_s - T_\infty)$$

where h is the heat transfer coefficient, T_s is the surface temperature and T_∞ is the fluid temperature.
Correlations in terms of the Nusselt number are often used to determine h:

$$Nu = \frac{hL_c}{k} \qquad h = \frac{kNu}{L_c}$$

where L_c is a characteristic length associated with the fluid flow geometry.

External Natural Convection: Horizontal Plate Up

Math Model

The heat transfer coefficient for laminar flow, $10^4 \lesssim Ra_L \lesssim 10^7$, from a hot plate, $T_s > T_\infty$, is (see Equation (9.30), p. 578, in [BLID11]):

$$\overline{Nu}_L = 0.54 Ra_L^{1/4} \quad (1)$$

and for turbulent flow, $10^7 \lesssim Ra_L \lesssim 10^{11}$, from a hot plate is (see Equation (9.31), p. 578, in [BLID11]):

$$\overline{Nu}_L = 0.15 Ra_L^{1/3} \quad (2)$$

External Natural Convection: Horizontal Plate Up

Math Model

For a cold plate, $T_s < T_\infty$, $10^4 \lesssim Ra_L \lesssim 10^9$, the correlation is (see Equation (9.32), p. 578, in [BLID11]):

$$\overline{Nu}_L = 0.52 Ra_L^{1/5} \tag{3}$$

Then the Rayleigh number, Ra_L, where $L = A/P$, is:

$$Ra_L = GrPr = \frac{g\rho^2 c\beta L^3 \left(T_s - T_\infty\right)}{k\mu} = \frac{g\beta L^3 \left(T_s - T_\infty\right)}{\nu\alpha} \tag{4}$$

Note that the fluid properties are evaluated at the film temperature, T_f:

$$T_f = \frac{T_s + T_\infty}{2} \tag{5}$$

External Natural Convection: Horizontal Plate Down

Math Model

The heat transfer coefficient for laminar flow, $10^4 \lesssim Ra_L \lesssim 10^7$, from a cold plate, $T_s < T_\infty$, is (see Equation (9.30), p. 578, in [BLID11]):

$$\overline{Nu}_L = 0.54 Ra_L^{1/4} \qquad (6)$$

and for turbulent flow, $10^7 \lesssim Ra_L \lesssim 10^{11}$, from a cold plate is (see Equation (9.31), p. 578, in [BLID11]):

$$\overline{Nu}_L = 0.15 Ra_L^{1/3} \qquad (7)$$

External Natural Convection: Horizontal Plate Down

Math Model

For a hot plate, $T_s > T_\infty$, $10^4 \lesssim Ra_L \lesssim 10^9$, the correlation is (see Equation (9.32), p. 578, in [BLID11]):

$$\overline{Nu}_L = 0.52 Ra_L^{1/5} \tag{8}$$

Then the Rayleigh number, Ra_L, where $L = A/P$, is:

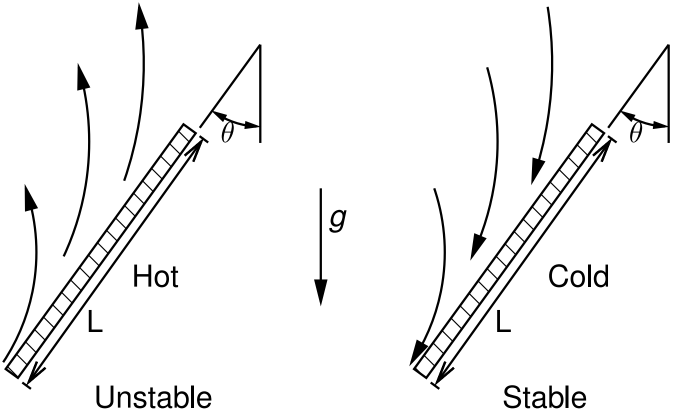
$$Ra_L = GrPr = \frac{g\rho^2 c\beta L^3 \left(T_s - T_\infty\right)}{k\mu} = \frac{g\beta L^3 \left(T_s - T_\infty\right)}{\nu\alpha} \tag{9}$$

Note that the fluid properties are evaluated at the film temperature, T_f:

$$T_f = \frac{T_s + T_\infty}{2} \tag{10}$$

External Natural Convection: Inclined Plate Up

Math Model



Stable Case, $T_s < T_\infty$:

The Nusselt number correlation for natural convection flow from a vertical plate (see Equation (9.26) and Equation (9.27), p. 573 in [BLID11] or [CC75]), with Rayleigh number:

$$Ra_L = GrPr = \frac{(g\cos\theta)\rho^2 c\beta L^3\left(T_s - T_\infty\right)}{k\mu}$$

Note that the fluid properties are evaluated at the film temperature, T_f:

$$T_f = \frac{T_s + T_\infty}{2}$$

Unstable Case, $T_s > T_\infty$:

The approach of Raithby and Hollands [RH98] is used. In this approach the heat transfer coefficient is evaluated for both a vertical plate with $g \cos \theta$ (see the stable case) and a horizontal plate with $g \cos(90 - \theta)$. The maximum of the two is then used.

Enclosure Radiation: The Exchange Factor, $\mathscr{F}$

Math Model

The exchange factor concept is based on proposing that there is a parameter, $\mathscr{F}_{ij}$, based on surface properties and enclosure geometry, that determines the radiative heat exchange between two surfaces [Hot54, HS67] :

$$Q_{ij} = A_i \mathscr{F}_{ij} \sigma (T_i^4 - T_j^4)$$

$\mathscr{F}$ is known by many names in the literature: script-F, gray body configuration factor, transfer factor and Hottel called it the over-all interchange factor.

For an enclosure with N surfaces, the net heat flow rate, Q_i, for surface i, is:

$$Q_i = \sum_{j=1}^{N} A_i \mathscr{F}_{ij} \left(\sigma T_i^4 - \sigma T_j^4 \right) = \sum_{j=1}^{N} A_i \mathscr{F}_{ij} \left(E_{bi} - E_{bj} \right)$$

Exchange Factor, $\mathscr{F}$, Properties

Math Model

Reciprocity:

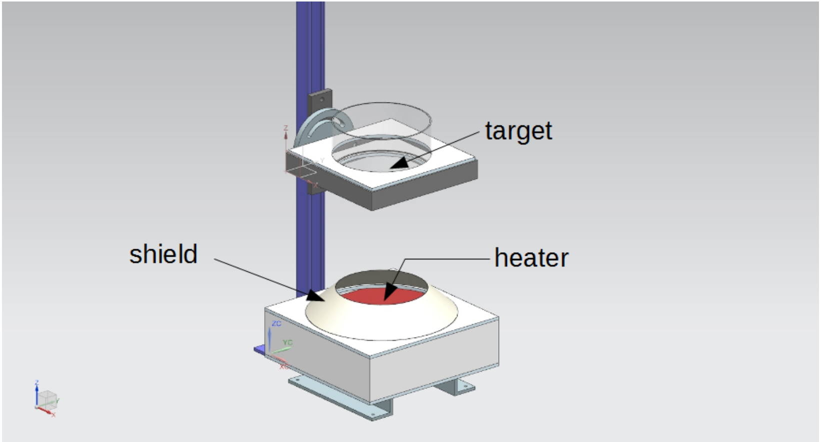
$$A_i \mathscr{F}_{ij} = A_j \mathscr{F}_{ji}$$

Summation:

$$\sum_{j=1}^{N} \mathscr{F}_{ij} = \epsilon_i$$

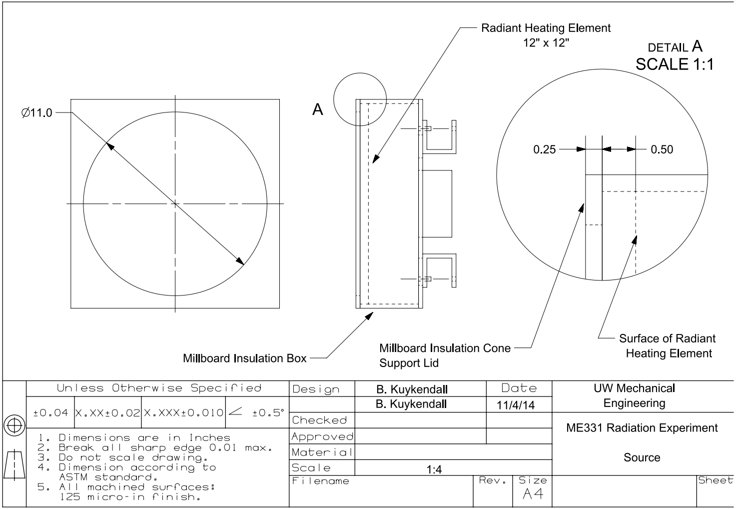
Radiation Heat Transfer Experiment

Geometry



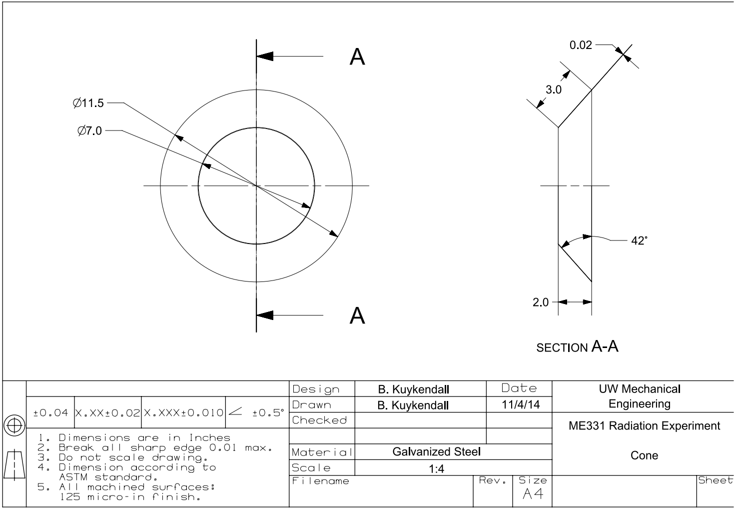
Heater

Geometry

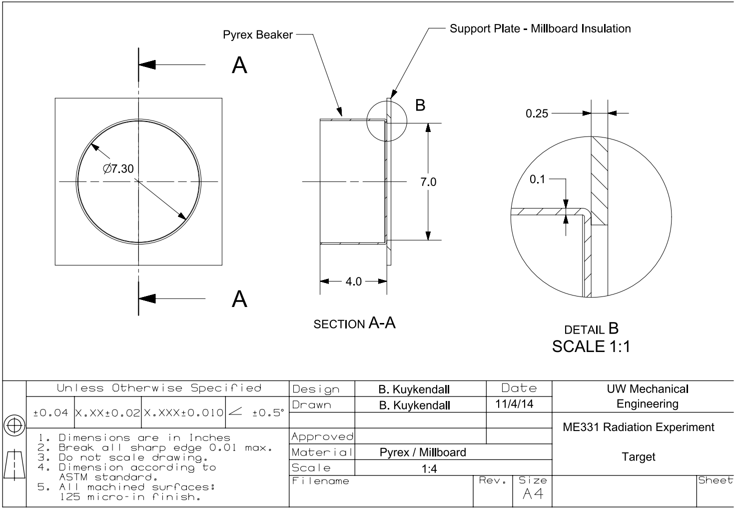


Shield

Geometry



Target
Geometry



Measurements

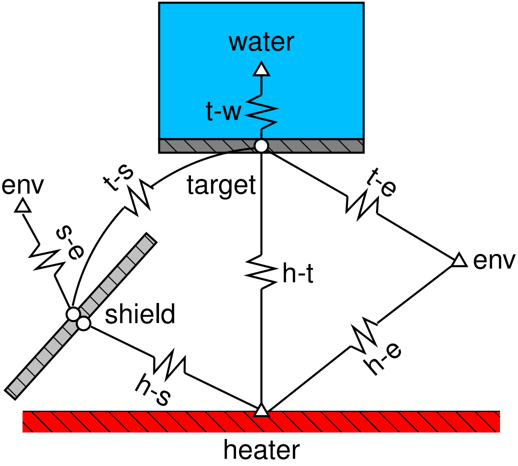
Test Data

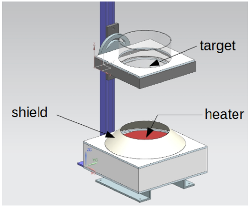
For each target material:

- Heater temperature
- Water evaporation rate
- Ambient air temperature
- Distance from target to heater

TNSolver Thermal Network Model

TNSolver Input File





External Natural Convection (ENC): Horizontal Plate Up

TNSolver Input File

$$Q_{ij} = hA(T_s - T_\infty)$$

Heat transfer coefficient, h, is evaluated using the correlation for external natural convection from horizontal plate facing up.

```
Begin Conductors
!                  Ts   Tinf
! label    type    nd_i nd_j parameters
   (S) ENChplateup (S)  (S)  (S)  (R)  (R)  ! mat, L=A/P, A

End Conductors
```

Note that Ra, Nu and h are reported in the output file.

External Natural Convection (ENC): Horizontal Plate Down

TNSolver Input File

$$Q_{ij} = hA(T_s - T_\infty)$$

Heat transfer coefficient, h, is evaluated using the correlation for external natural convection from horizontal plate facing down.

```
Begin Conductors
!                       Ts   Tinf
! label    type      nd_i nd_j parameters
  (S) ENChplatedown (S) (S) (S) (R) (R) ! mat, L=A/P, A

End Conductors
```

Note that Ra, Nu and h are reported in the output file.

External Natural Convection (ENC): Inclined Plate Up

TNSolver Input File

$$Q_{ij} = hA(T_s - T_\infty)$$

Heat transfer coefficient, h, is evaluated using the correlation for external natural convection from horizontal plate facing down.

```
Begin Conductors
!                 Ts  Tinf
! label    type   nd_i nd_j parameters
  (S) ENCiplateup (S)  (S) (S) (R) (R) (R) ! mat, L, angle, A

End Conductors
```

Note that Ra, Nu and h are reported in the output file.

Radiation Enclosure

TNSolver Input File

Each radiation enclosure is described by the surface labels, emissivities, areas and the view factor matrix $[F]$:

```
Begin Radiation Enclosure

! label   emiss   area   view factor matrix entries
    (S)     (R)    (R)    (R ...)

End Radiation Enclosure
```

The generated radiation conductors are reported in the output file.

Example Input File

TNSolver Input File

```
Begin Solution Parameters

  title = Radiation Heat Transfer Experiment - Black Target
  type                         = steady
  nonlinear convergence        = 1.0e-8
  maximum nonlinear iterations = 50

End Solution Parameters

Begin Conductors

! Conduction through the beaker wall, 0.1" thick pyrex glass
     t-bbin  conduction targ bbin  0.14 0.00254 0.024829 ! k L A
! Convection from beaker to water
! label    type        nd_i   nd_j    mat     L      A
  t-w  ENChplateup    bbin   water  water 0.04445 0.02483
! Convection from target to air
! label    type         nd_i   nd_j    mat     L      A
  t-air ENChplatedown targ  env     air  0.0889 0.0248
! Convection from outer shield to air
! label    type         nd_i   nd_j  mat    L   theta   A
  s-air  ENCiplateup  s_out  env  air 0.0508 48.0 0.056439
! Conduction from inner to outer side of shield
  shield  conduction  s_in s_out  steel  0.001 0.056439

End Conductors
```

Example Input File (continued)

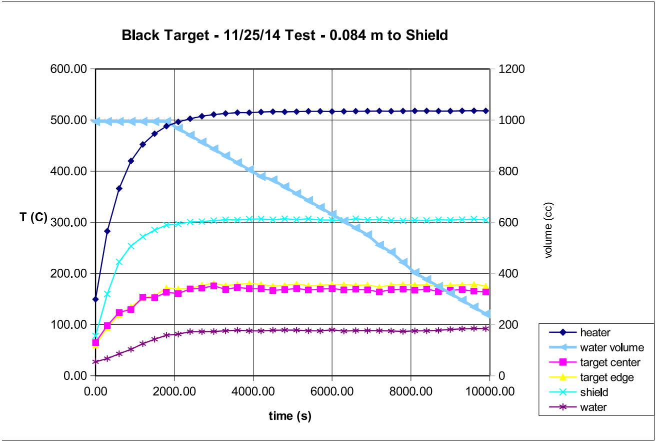
TNSolver Input File

```
Begin Radiation Enclosure

! surf emiss  A      Fij
 htr   0.92 0.06701 0.0     0.1264  0.68415 0.0    0.1893
 targ  0.95 0.02482 0.34132 0.0     0.00603 0.1031 0.5494
 s_in  0.28 0.05643 0.81231 0.00265 0.12278 0.0    0.0622
 s_out 0.28 0.05643 0.0     0.0453  0.0     0.0    0.9546
 env   1.0  0.1184  0.10711 0.1151  0.02965 0.4547 0.2933

End Radiation Enclosure

Begin Boundary Conditions

!  type        Tb      Node(s)
  fixed_T     23.0      env
  fixed_T     88.3      water
  fixed_T    515.0      htr

End Boundary Conditions
```

Black Target Test Data

Results

Heater Emissivity = 0.92 @ 500 C
Target Emissivity = 0.95 (black) @ 150 C



Black Target Water Evaporation Rate

Results

MATLAB: polyfit(time, vol, 1)
Evaporation Rate = 0.093793E-6 m^3/s
Water density (at 88.3 C) $\rho = 967\, kg/m^3$
Water latent heat of vaporization, $h_{fg} = 2,256,000\, J/kg$
Q = 967*0.093793E-6*2,256,000 = 205 W

Black Target View Factors, F_{ij}

Results

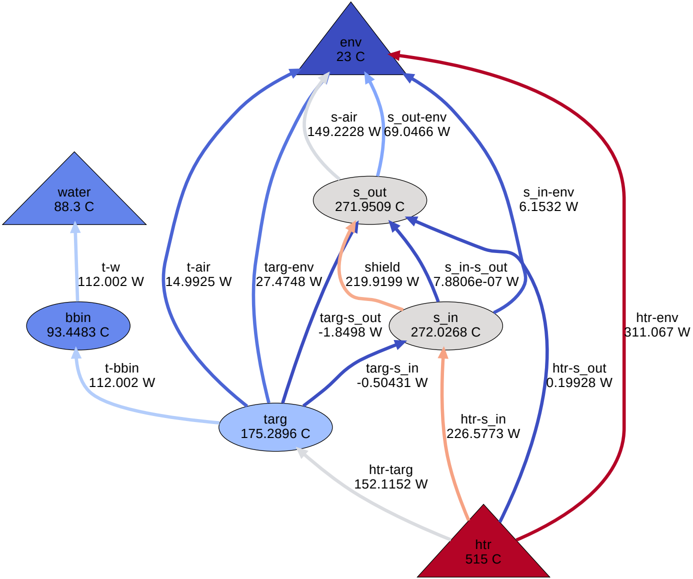
	Area (m^2)	Emissivity, ϵ
heater	0.06701	0.92
target	0.02483	0.95
shield	0.05644	0.28
env	0.1185	1.0

View Factor Matrix, $[F]$

	heater	target	s_in	s_out	env
heater	0	0.1265	0.6842	0	0.1894
target	0.3413	0	0.006039	0.1032	0.5494
s_in	0.8123	0.002657	0.1228	0	0.06224
s_out	0	0.0454	0	0	0.9546
env	0.1071	0.1151	0.02965	0.4547	0.2934

Thermal Network Solution

Results



Conclusion

- Math model components for heat transfer analysis
- TNSolver input file specifics
- Demonstrated simulation results for an experimental data set

Questions?

- All surfaces emit thermal radiation.
- The emittance, ϵ, is the ratio of actual energy emitted to that of a black surface at the same temperature.
- All radiation impinging on a surface will either be reflected, absorbed or transmitted.
- Reflectance or reflectivity, ρ, is the amount reflected.
- Absorptance or absorptivity, α, is the amount absorbed.
- Transmittance or transmissivity, τ, is the amount transmitted through the material.

Surface Properties for Radiation Heat Transfer (continued)

Appendix

Summation property for all incident radiation:

$$\rho + \alpha + \tau = 1$$

An opaque surface has $\tau = 0$, so $\rho + \alpha = 0$.

Kirchoff's law provides that $\epsilon = \alpha$ for gray, diffuse surfaces.

Diffuse is a modifier which means the property is not a function of direction.

Gray is a modifier indicating no dependence on wavelength.

Spectral is a modifier which means dependence on wavelength.

Specular is a modifier which means mirror-like reflection.

Characteristics of Real Surfaces

Appendix

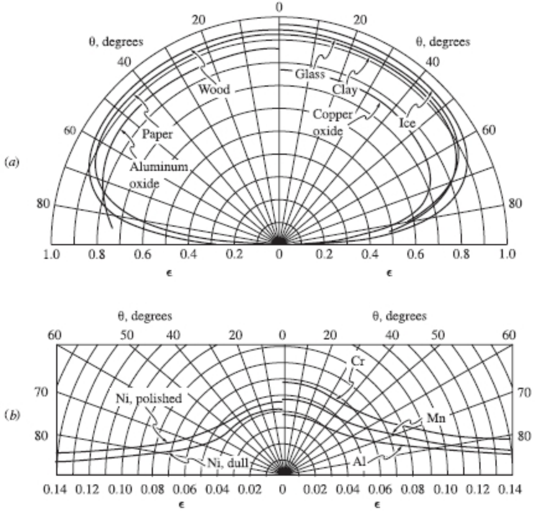


Figure 8.4 Directional variation of surface emittances: (*a*) for several nonmetals; (*b*) for several metals. (From Schmidt and Eckert, 1935.)

Figure borrowed from M.F. Modest, *Radiative Heat Transfer*, second edition, Academic Press, New York, 2003

View Factor Properties

Appendix

Summation Rule (Equation (13.4), page 830 in [BLID11]):

$$\sum_{i=1}^{N} F_{ij} = 1$$

Reciprocity Rule (Equation (13.3), page 829 in [BLID11]):

$$A_i F_{ij} = A_j F_{ji}$$

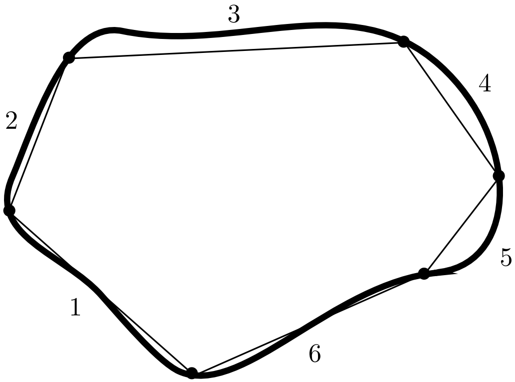
Addition of View Factors for Subdivided Surfaces (Equation (13.5), page 833 and Figure 13.7, page 835 in [BLID11]):

$$F_{i(j)} = \sum_{k=1}^{N} F_{ik}$$

Net Radiation Method for Enclosures

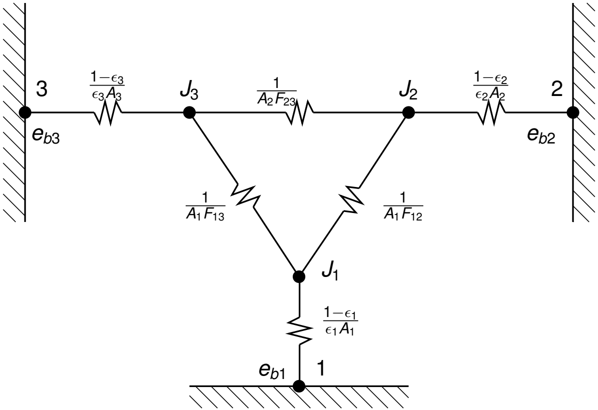
Appendix

- The enclosure geometry is approximated by a set of N ideal surfaces
  - Diffuse gray emission/absorption
  - Diffuse gray reflections
  - Each surface is isothermal, with uniform heat flux



Oppenheim Network for Three Surfaces

Appendix



A.K. Oppenheim, ”Radiation Analysis by the Network Method,” Transactions of the ASME, vol. 78, pp. 725–735, 1956

The Surface Radiosity

Appendix

Radiosity, J_i, for surface i is:

$$J_i = \underbrace{\epsilon_i \sigma T_i^4}_{\text{emmision}} + \underbrace{\rho_i G_i}_{\text{reflected irradiation}}$$

Surface irradiation, G_i is:

$$G_i = \sum_{i=1}^{N} F_{ij} J_j$$

Then noting that $\rho_i = 1 - \epsilon_i$, the equation for the radiosity from surface i is:

$$J_i - (1 - \epsilon_i)\left(\sum_{i=1}^{N} F_{ij} J_j\right) = \epsilon_i \sigma T_i^4$$

System of Equations for the Radiosity

Rearrangement of the radiosity equation for surface *i* leads to:

$$\sum_{i=1}^{N} \left[\delta_{ij} - (1 - \epsilon_i) F_{ij}\right] J_j = \epsilon_i \sigma T_i^4$$

Combining the radiosity equations for each surface in the enclosure leads to a system of equations:

$$[A] \{J\} = \{b\}$$

Where:

$$A_{ij} = \delta_{ij} - (1 - \epsilon_i) F_{ij}$$

$$b_i = \epsilon_i \sigma T_i^4$$

The surface temperature, T_i, is provided by the solution of the energy conservation equation.

Once the radiosity, J_i is known for a surface, the irradiation is evaluated:

$$G_i = \sum_{i=1}^{N} F_{ij} J_j$$

This is then used to evaluate the net radiant thermal energy for surface i:

$$q_i|_{radiation} = \epsilon_i \sigma T_i^4 - \alpha_i G_i$$

This heat flux boundary condition is then applied to the energy conservation equation solution so that updated surface temperatures can be determined.
Iteration between the energy equation and the radiosity is continued until convergence.

Matrix Form of the Radiosity Equation

Appendix

$$\sum_{i=1}^{N} \left[\delta_{ij} - (1 - \epsilon_i) F_{ij}\right] J_j = \epsilon_i \sigma T_i^4$$

$$([I] - [\rho][F]) \{J\} = [\epsilon] \{E_b\}$$

where, for $N = 3$:

$$\{J\} = \left\{ \begin{array}{c} J_1 \\ J_2 \\ J_3 \end{array} \right\} \qquad \left\{\sigma T^4\right\} = \left\{ \begin{array}{c} \sigma T_1^4 \\ \sigma T_2^4 \\ \sigma T_3^4 \end{array} \right\} = \{E_b\} = \left\{ \begin{array}{c} E_{b1} \\ E_{b2} \\ E_{b3} \end{array} \right\}$$

$$[\epsilon] = \begin{bmatrix} \epsilon_1 & 0 & 0 \\ 0 & \epsilon_2 & 0 \\ 0 & 0 & \epsilon_3 \end{bmatrix} \; [F] = \begin{bmatrix} F_{11} & F_{12} & F_{13} \\ F_{21} & F_{22} & F_{23} \\ F_{31} & F_{32} & F_{33} \end{bmatrix} \; [I] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$[\rho] = \begin{bmatrix} \rho_1 & 0 & 0 \\ 0 & \rho_2 & 0 \\ 0 & 0 & \rho_3 \end{bmatrix} = [I] - [\epsilon] = \begin{bmatrix} 1 - \epsilon_1 & 0 & 0 \\ 0 & 1 - \epsilon_2 & 0 \\ 0 & 0 & 1 - \epsilon_3 \end{bmatrix}$$

$$([I] - [\rho][F])\{J\} = [\epsilon]\{E_b\}$$

Using the definition of radiosity:

$$\{J\} = [\epsilon]\{E_b\} + [\rho]\{G\} = [\epsilon]\{E_b\} + [\rho][F]\{J\}$$

and the net surface heat flux ($\alpha = \epsilon$):

$$\{q\} = [\epsilon]\{E_b\} - [\alpha]\{G\} = [\epsilon]\{E_b\} - [\epsilon][F]\{J\}$$

then some algebraic manipulation leads to:

$$([I] - [F][\rho])[\epsilon]^{-1}\{q\} = ([I] - [F])\{E_b\}$$

or,

$$\{q\} = [\epsilon]([I] - [F][\rho])^{-1}([I] - [F])\{E_b\}$$

Exchange Factor, $\mathscr{F}$

Appendix

The exchange factor concept is based on proposing that there is a parameter, $\mathscr{F}_{ij}$, based on surface properties and enclosure geometry, that determines the radiative heat exchange between two surfaces [Hot54, HS67] :

$$Q_{ij} = A_i \mathscr{F}_{ij} \sigma (T_i^4 - T_j^4)$$

$\mathscr{F}$ is known by many names in the literature: script-F, gray body configuration factor, transfer factor and Hottel called it the over-all interchange factor.

For an enclosure with N surfaces, the net heat flow rate, Q_i, for surface i, is:

$$Q_i = \sum_{j=1}^{N} A_i \mathscr{F}_{ij} \left(\sigma T_i^4 - \sigma T_j^4 \right) = \sum_{j=1}^{N} A_i \mathscr{F}_{ij} \left(E_{bi} - E_{bj} \right)$$

Exchange Factor, $\mathscr{F}$, Properties

Appendix

Reciprocity:

$$A_i \mathscr{F}_{ij} = A_j \mathscr{F}_{ji}$$

Summation:

$$\sum_{j=1}^{N} \mathscr{F}_{ij} = \epsilon_i$$

Matrix Form of Enclosure Radiation with $\mathscr{F}$

Appendix

$$Q_i = \sum_{j=1}^{N} A_i \mathscr{F}_{ij} \left(E_{bi} - E_{bj} \right)$$

$$\{Q\} = [A] \left([\epsilon] - [\mathscr{F}] \right) \{E_b\}$$

$$\{q\} = [A]^{-1} \{Q\} = \left([\epsilon] - [\mathscr{F}] \right) \{E_b\}$$

where, for $N = 3$:

$$\{Q\} = \begin{Bmatrix} Q_1 \\ Q_2 \\ Q_3 \end{Bmatrix} \quad [A] = \begin{bmatrix} A_1 & 0 & 0 \\ 0 & A_2 & 0 \\ 0 & 0 & A_3 \end{bmatrix}$$

$$[\mathscr{F}] = \begin{bmatrix} \mathscr{F}_{11} & \mathscr{F}_{12} & \mathscr{F}_{13} \\ \mathscr{F}_{21} & \mathscr{F}_{22} & \mathscr{F}_{23} \\ \mathscr{F}_{31} & \mathscr{F}_{32} & \mathscr{F}_{33} \end{bmatrix} \quad \{E_b\} = \begin{Bmatrix} \sigma T_1^4 \\ \sigma T_2^4 \\ \sigma T_3^4 \end{Bmatrix} \quad [\epsilon] = \begin{bmatrix} \epsilon_1 & 0 & 0 \\ 0 & \epsilon_2 & 0 \\ 0 & 0 & \epsilon_3 \end{bmatrix}$$

Exchange Factors, $\mathscr{F}$, from View Factors, F

Appendix

The net heat flux using view factors, F_{ij}, is:

$$\{q\} = [\epsilon]([I] - [F][\rho])^{-1}([I] - [F])\{E_b\}$$

The net heat flux using exchange factors, $\mathscr{F}_{ij}$, is:

$$\{q\} = ([\epsilon] - [\mathscr{F}])\,\{E_b\}$$

The two enclosure heat fluxes are equal, so equating gives:

$$([\epsilon] - [\mathscr{F}])\,\{E_b\} = [\epsilon]([I] - [F][\rho])^{-1}([I] - [F])\{E_b\}$$

$$([\epsilon] - [\mathscr{F}]) = [\epsilon]([I] - [F][\rho])^{-1}([I] - [F])$$

$$[\mathscr{F}] = [\epsilon]\left([I] - ([I] - [F][\rho])^{-1}([I] - [F])\right)$$

See [IB63] for an early reference to this method.

Linearization of Radiation Conductors

Appendix

The temperature is linearized using a two term Taylor series expansion about the previous iteration temperature, T^*:

$$T_i^4 \approx (T_i^*)^4 + (T_i - T_i^*)\, 4(T_i^*)^3$$

$$T_i^4 \approx 4(T_i^*)^3 T_i - 3(T_i^*)^4$$

$$T_j^4 \approx \left(T_j^*\right)^4 + \left(T_j - T_j^*\right) 4\left(T_j^*\right)^3$$

$$T_j^4 \approx 4\left(T_j^*\right)^3 T_j - 3\left(T_j^*\right)^4$$

The linearized form of the heat transfer rate is:

$$Q_{ij} = \sigma \mathscr{F}_{ij} A_i \left[4(T_i^*)^3 T_i - 3(T_i^*)^4 - 4\left(T_j^*\right)^3 T_j + 3\left(T_j^*\right)^4\right]$$

$$Q_{ij} = \sigma \mathscr{F}_{ij} A_i \left\{4(T_i^*)^3 T_i - 4\left(T_j^*\right)^3 T_j\right\} - \sigma \mathscr{F}_{ij} A_i \left\{3(T_i^*)^4 - 3\left(T_j^*\right)^4\right\}$$

References I

[BLID11] T.L. Bergman, A.S. Lavine, F.P. Incropera, and D.P. DeWitt.
Introduction to Heat Transfer.
John Wiley & Sons, New York, sixth edition, 2011.

[CC75] Stuart W. Churchill and Humbert H.S. Chu.
Correlating equations for laminar and turbulent free convection from a vertical plate.
International Journal of Heat and Mass Transfer, 18(11):1323–1329, 1975.

[Hot54] H. C. Hottel.
Radiant-heat transmission.
In *Heat Transmission* [McA54], chapter 4, pages 55–125.

References II

[HS67] H. C. Hottel and A. F. Sarofim.
Radiative Transfer.
McGraw-Hill, New York, 1967.

[IB63] T. Ishimoto and J. T. Bevans.
Method of evaluating script f for radiant exchange within an enclosure.
AIAA Journal, 1(6):1428–1429, 1963.

[McA54] W. H. McAdams.
Heat Transmission.
McGraw-Hill, New York, third edition, 1954.

References III

[RH98] G. D. Raithby and K. G. T. Hollands.
Natural convection.
In Warren M. Rohsenow, James R Hartnett, and Young I. Cho, editors, *Handbook of Heat Transfer*, chapter four, pages 4.1–4.99. McGraw-Hill, New York, third edition, 1998.