Radiation Heat Transfer Experiment

Thermal Network Solution with TNSolver

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Outline

- Math Model
- Geometry
- Test Data
- Thermal Network Model Solution with TNSolver
- Results

Surface Properties for Radiation Heat Transfer

Math Model

- All surfaces emit thermal radiation.
- The emittance, ϵ, is the ratio of actual energy emitted to that of a black surface at the same temperature.
- All radiation impinging on a surface will either be reflected, absorbed or transmitted.
- Reflectance or reflectivity, ρ, is the amount reflected.
- Absorptance or absorptivity, α, is the amount absorbed.
- Transmittance or transmissivity, τ, is the amount transmitted through the material.

Surface Properties for Radiation Heat Transfer (continued)

Math Model

Summation property for all incident radiation:

$$\rho + \alpha + \tau = 1$$

An opaque surface has $\tau = 0$, so $\rho + \alpha = 0$.

Kirchoff's law provides that $\epsilon = \alpha$ for gray, diffuse surfaces.

Diffuse is a modifier which means the property is not a function of direction.

Gray is a modifier indicating no dependence on wavelength.

Spectral is a modifier which means dependence on wavelength.

Specular is a modifier which means mirror-like reflection.

Characteristics of Real Surfaces

Math Model

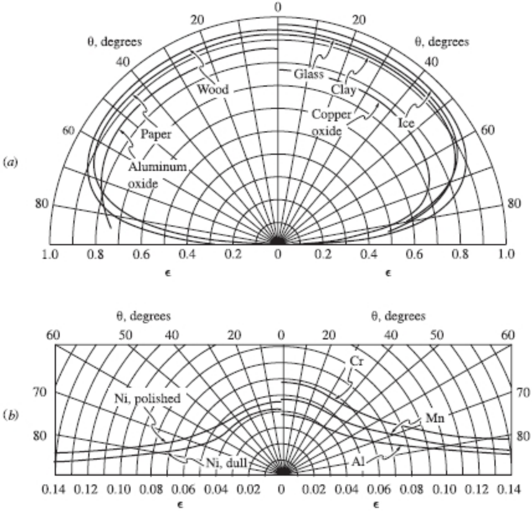


Figure 8.4 Directional variation of surface emittances: (*a*) for several nonmetals; (*b*) for several metals. (From Schmidt and Eckert, 1935.)

Figure borrowed from M.F. Modest, *Radiative Heat Transfer*, second edition, Academic Press, New York, 2003

View Factor Properties

Math Model

Summation Rule (Equation (13.4), page 830 in [BLID11]):

$$\sum_{i=1}^{N} F_{ij} = 1$$

Reciprocity Rule (Equation (13.3), page 829 in [BLID11]):

$$A_i F_{ij} = A_j F_{ji}$$

Addition of View Factors for Subdivided Surfaces (Equation (13.5), page 833 and Figure 13.7, page 835 in [BLID11]):

$$F_{i(j)} = \sum_{k=1}^{N} F_{ik}$$

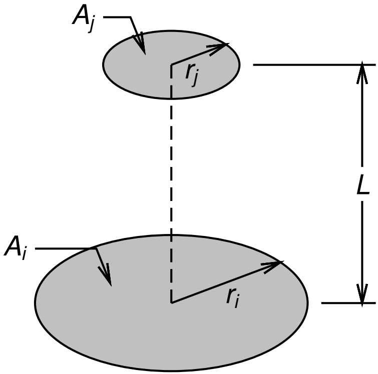
View Factor for Coaxial Parallel Disks

Math Model

$$R_i = \frac{r_i}{L}, \; R_j = \frac{r_j}{L}$$

$$S = 1 + \frac{1 + R_j^2}{R_i^2}$$

$$F_{ij} = \frac{1}{2}\left\{ S - \sqrt{S^2 - 4\left(\frac{r_j}{r_i}\right)^2} \right\}$$



See Table 13.2, page 833 in [BLID11] or Table 10.3, page 546 in [LL12]
Also see the web site: A Catalog of Radiation Heat Transfer Configuration Factors, by John R. Howell, specifically C-41: Disk to parallel coaxial disk of unequal radius.

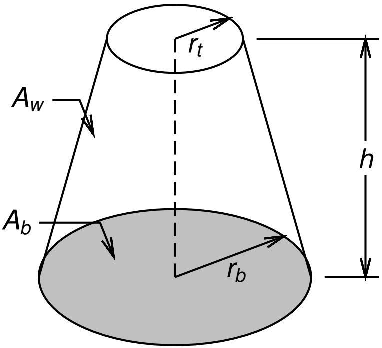
View Factor for Conical Frustum Wall to its Base

Math Model

$$H = \frac{h}{r_b} \quad R = \frac{r_b}{r_t}$$

$$X = 1 + R^2 + H^2$$

$$F_{wb} = \frac{2R^2 - X + \sqrt{X^2 - 4R^2}}{2\,(1 + R)\sqrt{X - 2R}}$$



See the web site: A Catalog of Radiation Heat Transfer Configuration Factors, by John R. Howell, specifically C-112: Interior of frustum of right circular cone to base.

View Factor for Disk to Coaxial Cone

Math Model

$$\alpha = \tan^{-1}\frac{r_c}{h}$$

$$S = \frac{r_d}{L} \quad R = \frac{r_c}{r_d} \quad X = S + R\cot\alpha$$

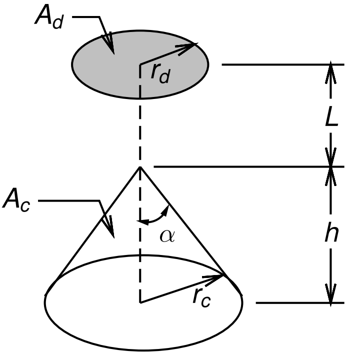
$$A = \sqrt{X^2 + (1+R)^2}$$

$$B = \sqrt{X^2 + (1-R)^2}$$

$$C = \sqrt{\cos\alpha + S\sin\alpha}$$

$$D = \sqrt{\cos\alpha - S\sin\alpha}$$

$$E = R\cot\alpha - S$$



See the web site: A Catalog of Radiation Heat Transfer Configuration Factors, by John R. Howell, specifically C-48: Disk to coaxial cone.

View Factor for Disk to Coaxial Cone (continued)

Math Model

For $\alpha \geq \tan^{-1}\frac{1}{S}$:

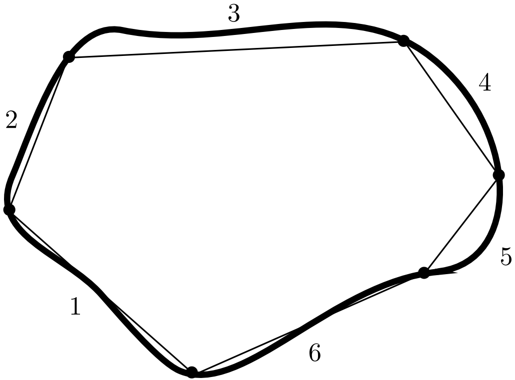
$$F_{dc} = \frac{1}{2}\left\{R^2 + X^2 + 1 - \sqrt{\left(1 + R^2 + X^2\right)^2 - 4R^2}\right\}$$

For $\alpha < \tan^{-1}\frac{1}{S}$:

$$F_{dc} = \frac{1}{\pi}\left\{-AB\tan^{-1}\frac{AC}{BD} + \left(1 + S^2\right)\tan^{-1}\frac{C}{D}\right.$$
$$+\frac{\sin\alpha}{\cos^2\alpha}\left[XE\tan^{-1}\frac{CD}{X} + S^2\tan^{-1}\frac{CD}{S} + (CD)^2\left(\tan^{-1}\frac{X}{CD} - \tan^{-1}\frac{S}{CD}\right)\right]$$
$$\left. + \left[\frac{R\,(X+S)}{\sin 2\alpha} - SR\tan\alpha\right]\cos^{-1}\left(-S\tan\alpha\right)\right\}$$

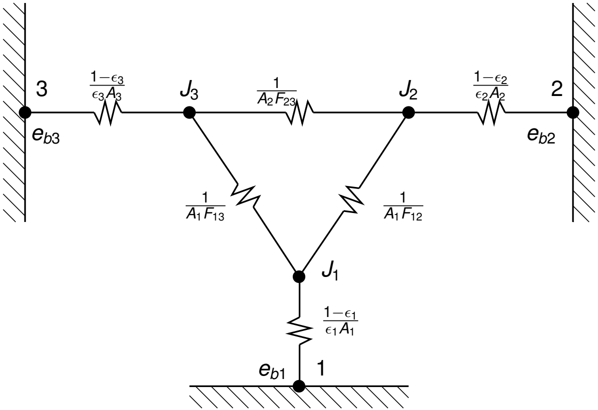
Net Radiation Method for Enclosures

- The enclosure geometry is approximated by a set of N ideal surfaces
  - Diffuse gray emission/absorption
  - Diffuse gray reflections
  - Each surface is isothermal, with uniform heat flux



Oppenheim Network for Three Surfaces

Math Model



A.K. Oppenheim, ”Radiation Analysis by the Network Method,” Transactions of the ASME, vol. 78, pp. 725–735, 1956

The Surface Radiosity

Math Model

Radiosity, J_i, for surface i is:

$$J_i = \underbrace{\epsilon_i \sigma T_i^4}_{\text{emmision}} + \underbrace{\rho_i G_i}_{\text{reflected irradiation}}$$

Surface irradiation, G_i is:

$$G_i = \sum_{i=1}^{N} F_{ij} J_j$$

Then noting that $\rho_i = 1 - \epsilon_i$, the equation for the radiosity from surface i is:

$$J_i - (1 - \epsilon_i) \left(\sum_{i=1}^{N} F_{ij} J_j \right) = \epsilon_i \sigma T_i^4$$

System of Equations for the Radiosity

Math Model

Rearrangement of the radiosity equation for surface i leads to:

$$\sum_{i=1}^{N} \left[\delta_{ij} - (1 - \epsilon_i) F_{ij}\right] J_j = \epsilon_i \sigma T_i^4$$

Combining the radiosity equations for each surface in the enclosure leads to a system of equations:

$$[A] \{J\} = \{b\}$$

Where:

$$A_{ij} = \delta_{ij} - (1 - \epsilon_i) F_{ij}$$

$$b_i = \epsilon_i \sigma T_i^4$$

The surface temperature, T_i, is provided by the solution of the energy conservation equation.

Application of Radiosity to Heat Conduction Equation

Math Model

Once the radiosity, J_i is known for a surface, the irradiation is evaluated:

$$G_i = \sum_{i=1}^{N} F_{ij} J_j$$

This is then used to evaluate the net radiant thermal energy for surface i:

$$q_i|_{radiation} = \epsilon_i \sigma T_i^4 - \alpha_i G_i$$

This heat flux boundary condition is then applied to the energy conservation equation solution so that updated surface temperatures can be determined.
Iteration between the energy equation and the radiosity is continued until convergence.

Matrix Form of the Radiosity Equation

Math Model

$$\sum_{i=1}^{N} \left[\delta_{ij} - (1 - \epsilon_i) F_{ij}\right] J_j = \epsilon_i \sigma T_i^4$$

$$([I] - [\rho][F]) \{J\} = [\epsilon] \{E_b\}$$

where, for $N = 3$:

$$\{J\} = \begin{Bmatrix} J_1 \\ J_2 \\ J_3 \end{Bmatrix} \qquad \left\{\sigma T^4\right\} = \begin{Bmatrix} \sigma T_1^4 \\ \sigma T_2^4 \\ \sigma T_3^4 \end{Bmatrix} = \{E_b\} = \begin{Bmatrix} E_{b1} \\ E_{b2} \\ E_{b3} \end{Bmatrix}$$

$$[\epsilon] = \begin{bmatrix} \epsilon_1 & 0 & 0 \\ 0 & \epsilon_2 & 0 \\ 0 & 0 & \epsilon_3 \end{bmatrix} \; [F] = \begin{bmatrix} F_{11} & F_{12} & F_{13} \\ F_{21} & F_{22} & F_{23} \\ F_{31} & F_{32} & F_{33} \end{bmatrix} \; [I] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$[\rho] = \begin{bmatrix} \rho_1 & 0 & 0 \\ 0 & \rho_2 & 0 \\ 0 & 0 & \rho_3 \end{bmatrix} = [I] - [\epsilon] = \begin{bmatrix} 1 - \epsilon_1 & 0 & 0 \\ 0 & 1 - \epsilon_2 & 0 \\ 0 & 0 & 1 - \epsilon_3 \end{bmatrix}$$

Enclosure Heat Flow from Radiosity

Math Model

$$([I] - [\rho][F])\{J\} = [\epsilon]\{E_b\}$$

Using the definition of radiosity:

$$\{J\} = [\epsilon]\{E_b\} + [\rho]\{G\} = [\epsilon]\{E_b\} + [\rho][F]\{J\}$$

and the net surface heat flux ($\alpha = \epsilon$):

$$\{q\} = [\epsilon]\{E_b\} - [\alpha]\{G\} = [\epsilon]\{E_b\} - [\epsilon][F]\{J\}$$

then some algebraic manipulation leads to:

$$([I] - [F][\rho])[\epsilon]^{-1}\{q\} = ([I] - [F])\{E_b\}$$

or,

$$\{q\} = [\epsilon]([I] - [F][\rho])^{-1}([I] - [F])\{E_b\}$$

Exchange Factor, $\mathscr{F}$

Math Model

The exchange factor concept is based on proposing that there is a parameter, $\mathscr{F}_{ij}$, based on surface properties and enclosure geometry, that determines the radiative heat exchange between two surfaces [Hot54, HS67] :

$$Q_{ij} = A_i \mathscr{F}_{ij} \sigma (T_i^4 - T_j^4)$$

$\mathscr{F}$ is known by many names in the literature: script-F, gray body configuration factor, transfer factor and Hottel called it the over-all interchange factor.
For an enclosure with N surfaces, the net heat flow rate, Q_i, for surface i, is:

$$Q_i = \sum_{j=1}^{N} A_i \mathscr{F}_{ij} \left(\sigma T_i^4 - \sigma T_j^4 \right) = \sum_{j=1}^{N} A_i \mathscr{F}_{ij} \left(E_{bi} - E_{bj} \right)$$

Exchange Factor, $\mathscr{F}$, Properties

Math Model

Reciprocity:

$$A_i \mathscr{F}_{ij} = A_j \mathscr{F}_{ji}$$

Summation:

$$\sum_{j=1}^{N} \mathscr{F}_{ij} = \epsilon_i$$

Matrix Form of Enclosure Radiation with $\mathscr{F}$

Math Model

$$Q_i = \sum_{j=1}^{N} A_i \mathscr{F}_{ij} \left(E_{bi} - E_{bj}\right)$$

$$\{Q\} = [A]\left([\epsilon] - [\mathscr{F}]\right)\{E_b\}$$

$$\{q\} = [A]^{-1}\{Q\} = \left([\epsilon] - [\mathscr{F}]\right)\{E_b\}$$

where, for $N = 3$:

$$\{Q\} = \left\{ \begin{array}{c} Q_1 \\ Q_2 \\ Q_3 \end{array} \right\} \quad [A] = \begin{bmatrix} A_1 & 0 & 0 \\ 0 & A_2 & 0 \\ 0 & 0 & A_3 \end{bmatrix}$$

$$[\mathscr{F}] = \begin{bmatrix} \mathscr{F}_{11} & \mathscr{F}_{12} & \mathscr{F}_{13} \\ \mathscr{F}_{21} & \mathscr{F}_{22} & \mathscr{F}_{23} \\ \mathscr{F}_{31} & \mathscr{F}_{32} & \mathscr{F}_{33} \end{bmatrix} \quad \{E_b\} = \left\{ \begin{array}{c} \sigma T_1^4 \\ \sigma T_2^4 \\ \sigma T_3^4 \end{array} \right\} \quad [\epsilon] = \begin{bmatrix} \epsilon_1 & 0 & 0 \\ 0 & \epsilon_2 & 0 \\ 0 & 0 & \epsilon_3 \end{bmatrix}$$

Exchange Factors, $\mathscr{F}$, from View Factors, F

Math Model

The net heat flux using view factors, F_{ij}, is:

$$\{q\} = [\epsilon]([I] - [F][\rho])^{-1}([I] - [F])\{E_b\}$$

The net heat flux using exchange factors, $\mathscr{F}_{ij}$, is:

$$\{q\} = ([\epsilon] - [\mathscr{F}])\,\{E_b\}$$

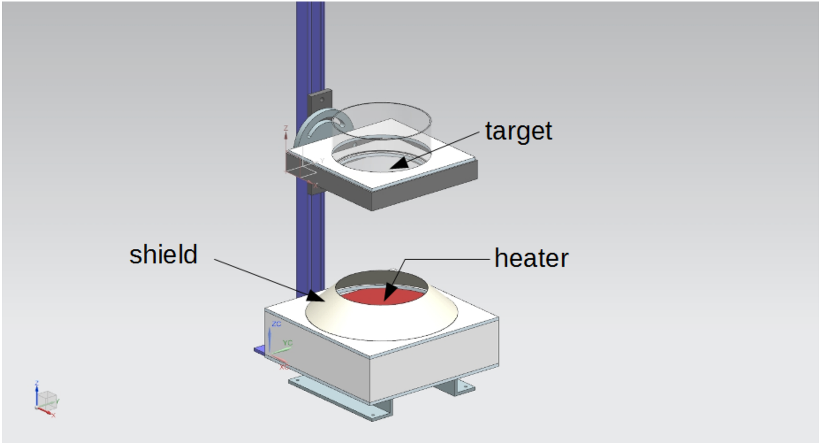
The two enclosure heat fluxes are equal, so equating gives:

$$([\epsilon] - [\mathscr{F}])\,\{E_b\} = [\epsilon]([I] - [F][\rho])^{-1}([I] - [F])\{E_b\}$$

$$([\epsilon] - [\mathscr{F}]) = [\epsilon]([I] - [F][\rho])^{-1}([I] - [F])$$

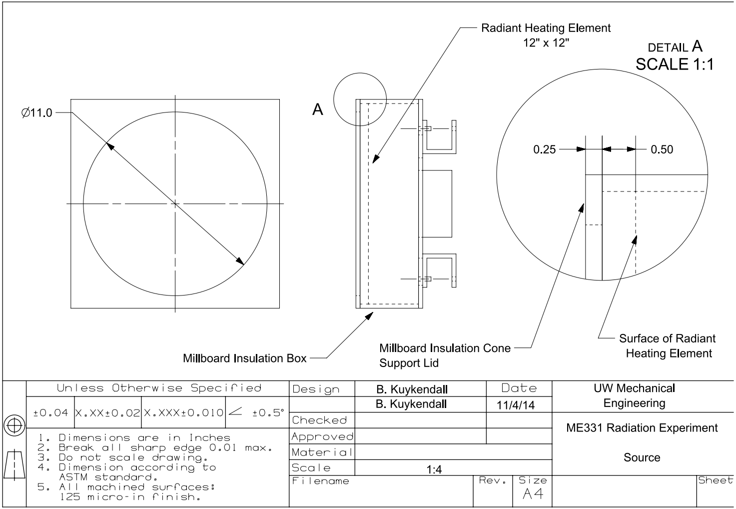
$$[\mathscr{F}] = [\epsilon]\left([I] - ([I] - [F][\rho])^{-1}([I] - [F])\right)$$

Radiation Heat Transfer Experiment
Geometry

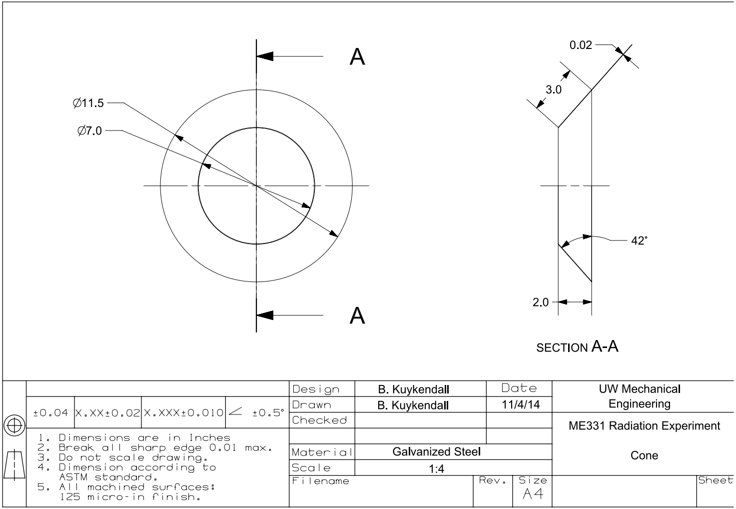


Heater

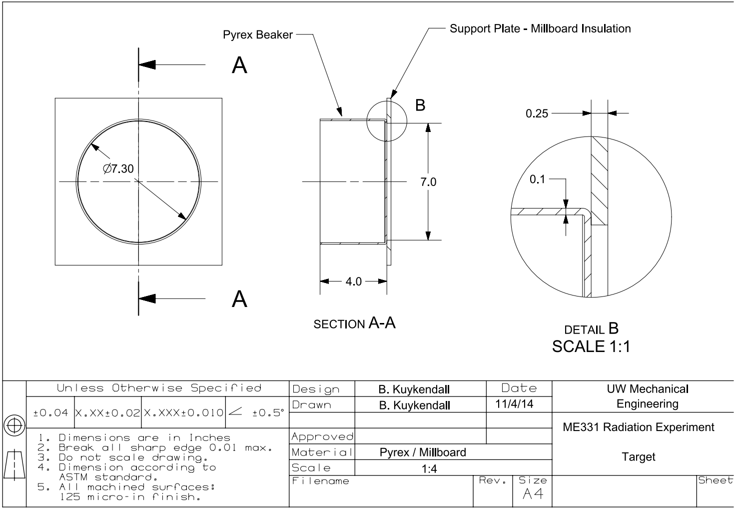
Geometry



Shield
Geometry



Target
Geometry



Surface Emissivity Material Properties

Geometry

Emissivity properties from Table A.8 in [BLID11]

Material	ϵ
Target: Alloy 1100 Aluminum, coated flat black	0.95
Target: 304 Stainless Steel	0.2 – 0.28
Heater: Ceran glass	0.8*
Shield: Galvanized steel	0.28

* To be confirmed

See [TD70] and [Gil02] for emissivity data, for a variety of materials.

Measurements

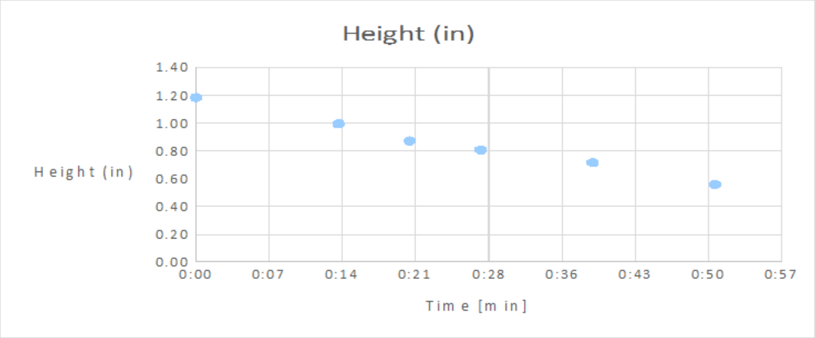
Test Data

For each target material:

- Heater temperature
- Water evaporation rate
- Ambient air temperature
- Distance from target to heater
- Shield in place or not

Sample Data Set
Test Data

Aluminum, Painted Flat Black, 4” Above The Heater



Heater Temperature was $\sim$ 525 C
Water Temperature was 100 C

Heat Transfer Analysis

Thermal Network Model

- Energy conservation: control volumes
- Identify and sketch out the control volumes
- Use the conductor analogy to represent energy transfer between the control volumes and energy generation or storage
  - Conduction, convection, radiation, other?
  - Capacitance
  - Sources or sinks
- State assumptions and determine appropriate parameters for each conductor
  - Geometry, material properties, etc.
- Which conductor(s)/source(s)/capacitance(s) are important to the required results?
  - Sensitivity analysis
- What is missing from the model? - peer/expert review

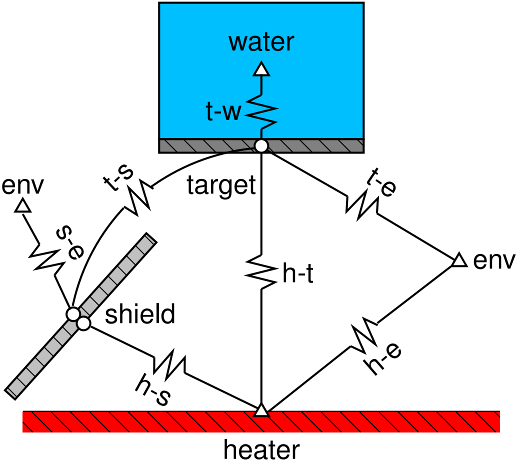
Thermal Network Terminology

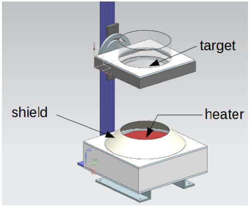
Thermal Network Model

- Geometry
  - Control Volume
    - Volume property, $V = \int_V dV$
    - Node: ●, $T_{\text{node}} = \int_V T(x_i) dV$
  - Control Volume Surface
    - Area property, $A = \int_A dA$
    - Surface Node: ○, $T_{\text{surface node}} = \int_A T(x_i) dA$
- Material properties
- Conductors
  - Conduction, convection, radiation
- Boundary conditions
  - Boundary node: ▲

Network Model

Thermal Network Model





Radiation Conductors

Thermal Network Model

$$Q_{ij} = \sigma \mathscr{F}_{ij} A_i (T_i^4 - T_j^4)$$

```
Begin Conductors

! label   type        nd_i    nd_j    script-F    A
   h-t   radiation   heater  target   0.0113   0.0613
   h-s   radiation   heater  shield   0.1711   0.0613
   h-e   radiation   heater  env      0.3076   0.0613
   t-s   radiation   target  shield   0.0042   0.0248
   t-e   radiation   target  env      0.0651   0.0248
   s-e   radiation   shield  env      0.0648   0.0516

End Conductors
```

Note that h_r is reported in the output file.

Linearization of Radiation Conductors

Thermal Network Model

The temperature is linearized using a two term Taylor series expansion about the previous iteration temperature, T^*:

$$T_i^4 \approx (T_i^*)^4 + (T_i - T_i^*)\,4(T_i^*)^3$$

$$T_i^4 \approx 4(T_i^*)^3 T_i - 3(T_i^*)^4$$

$$T_j^4 \approx \left(T_j^*\right)^4 + \left(T_j - T_j^*\right) 4\left(T_j^*\right)^3$$

$$T_j^4 \approx 4\left(T_j^*\right)^3 T_j - 3\left(T_j^*\right)^4$$

The linearized form of the heat transfer rate is:

$$Q_{ij} = \sigma \mathscr{F}_{ij} A_i \left[4(T_i^*)^3 T_i - 3(T_i^*)^4 - 4\left(T_j^*\right)^3 T_j + 3\left(T_j^*\right)^4\right]$$

$$Q_{ij} = \sigma \mathscr{F}_{ij} A_i \left\{4(T_i^*)^3 T_i - 4\left(T_j^*\right)^3 T_j\right\} - \sigma \mathscr{F}_{ij} A_i \left\{3(T_i^*)^4 - 3\left(T_j^*\right)^4\right\}$$

Using the Function disk2disk.m

Thermal Network Model

```
>> help disk2disk
 [F12, A1, F21, A2] = disk2disk(r1, r2, h)

   Description:

     View factor between two coaxial disks.

   Inputs:

     r1 = radius of upper disk
     r2 = radius of lower disk
     h  = distance between the disks

   Outputs:

     F12 = view factor from upper disk to lower disk
     A1  = area of the upper disk
     F21 = viewfactor from lower disk to upper disk
     A2  = area of the lower disk

   Reference:

     A Catalog of Radiation Heat Transfer Configuration Factors,
     third edition, John R. Howell, The University of Texas at Austin

       http://www.thermalradiation.net/sectionc/C-41.html
```

Using the Function `disk2disk.m`

Thermal Network Model

$r_1 = 0.0889m$, $r_2 = 0.0889m$ and $h = 0.2m$

```
>>[F12, A1, F21, A2] = disk2disk(0.0889, 0.0889, 0.2)
F12 =
    0.1446
A1 =
    0.0248
F21 =
    0.1446
A2 =
    0.0248
```

Using the Function `cone2base.m`

Thermal Network Model

```
>> help cone2base
 [F12, A1, F21, A2] = cone2base(rb, rt, h)

   Description:

     View factor between the interior wall of a conical frustum and
     its base.

   Inputs:

     rb = radius of base
     rt = radius of top of conical frustum
     h  = height of the conical frustrum

   Outputs:

     F12 = view factor from interior wall to base
     A1  = area of the interior wall
     F21 = viewfactor from base to interior wall
     A2  = area of the base

   Reference:

     A Catalog of Radiation Heat Transfer Configuration Factors,
     third edition, John R. Howell, The University of Texas at Austin

       http://www.thermalradiation.net/sectionc/C-111.html
```

Using the Function `cone2base.m`

Thermal Network Model

$r_b = 0.1397m$, $r_t = 0.0889m$ and $h = 0.0508m$

```
>> [F12, A1, F21, A2]=cone2base(0.1397,0.0889,0.0508)
F12 =
    0.7872
A1 =
    0.0516
F21 =
    0.6624
A2 =
    0.0613
```

View Factor Matrix, $[F]$

Thermal Network Model

	A (m^2)
heater	0.0670
target	0.0248
shield	0.0564
env	0.1

A good rule of thumb when setting the area for radiation to the surroundings, is to make it about the same order of magnitude as the enclosure area.

	heater	target	shield	env
heater	0	0.1077	0.6851	0.2072
target	0.2907	0	0.0965	0.6127
shield	0.8134	0.0425	0	0.1441
env	0.1388	0.1521	0.0813	0.6277

Using the Function `scriptF.m`

Thermal Network Model

```
>>help scriptF
[sF] = scriptF(emiss, F)

   Description:

     Evaluate the "script-F" or transfer factors for an enclosure radiation
     problem, given the geometric view factors and surface emissivities.

   Input:

     emiss(:) = vector of surface emissivities
                  - length(emiss) = number of enclosure surfaces
     F(:,:)   = geometric view factor matrix
                  - can be a full or sparse matrix

   Output:

     sF(:,:)  = script-F matrix
                  - if F is sparse, then sF will be sparse
```

Using the Function `scriptF.m`

Thermal Network Model

```
>>emiss =  [0.8, 0.1, 0.28, 1.0];
>>F = [         0,    0.1077,     0.6851,     0.2072;
           0.2907,         0,     0.0965,     0.6127;
           0.8134,    0.0425,          0,     0.1441;
           0.1388,    0.1521,     0.0813,     0.6277 ]
>>[sF] = scriptF(emiss, F);
```

Exchange Factor Matrix, $[\mathscr{F}]$

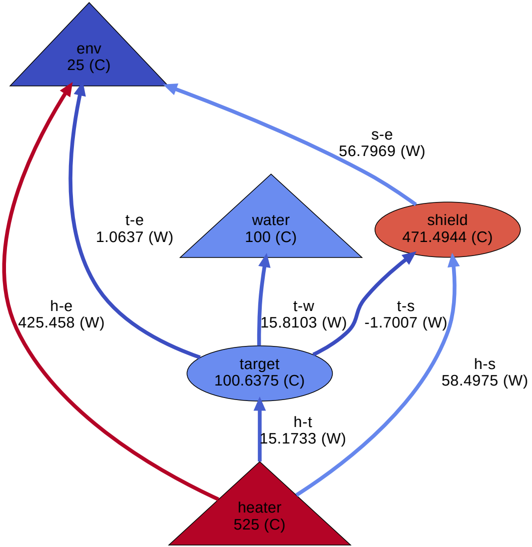
Thermal Network Model

	ϵ
heater	0.8
target	0.1
shield	0.28
env	1.0

	heater	target	shield	env
heater	0.3100	0.0113	0.1711	0.3076
target	0.0306	0.0001	0.0042	0.0651
shield	0.2031	0.0018	0.0102	0.0648
env	0.2061	0.0162	0.0366	0.7411

Thermal Network Solution

Results



Conclusion

- Provided view factor formulations required for the experiment
- Derived the relationship between the exchange factor, $\mathscr{F}$, and the Oppenheim network
- Developed the thermal network model of the radiation heat transfer experiment
- Demonstrated simulation results for preliminary experimental data

Questions?

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