Lumped Mass Heat Transfer Experiment

Thermal Network Solution with TNSolver

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Outline

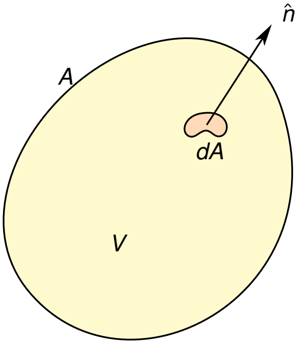
- Math Model
- TNSolver Input File
- Test Data Analysis

The Control Volume Concept

Math Model

$$\sum \text{Energy In} - \sum \text{Energy Out} =$$
$$\text{Energy Stored, Generated and/or Consumed}$$

Heat (transfer) is thermal energy transfer due to a temperature difference



The Lumped Capacitance Method: Biot Number

Math Model

The Biot number, Bi, is:

$$Bi = \frac{hL_c}{k} < 0.1 \qquad L_c = \frac{\text{volume}}{\text{surface area}} = \frac{V}{A}$$

where the characteristic length, L_c, is:

Brick	Cylinder	Sphere
$\frac{HWL}{2(HW+LH+WL)}$	$\frac{\pi r^2 L}{2\pi r^2+2\pi rL} = \frac{DL}{2(D+2L)}$	$\frac{(4/3)\pi r^3}{4\pi r^2} = \frac{D}{6}$

The heat flow rate is:

$$Q = hA(T_s - T_\infty)$$

where h is the heat transfer coefficient, T_s is the surface temperature and T_∞ is the fluid temperature.
Correlations in terms of the Nusselt number are often used to determine h:

$$Nu = \frac{hL_c}{k} \qquad h = \frac{kNu}{L_c}$$

where L_c is a characteristic length associated with the fluid flow geometry.

External Forced Convection over a Sphere

Math Model

Equation (7.48), p. 444, in [BLID11]

$$\overline{Nu}_D = 2 + \left(0.4 Re_D^{1/2} + 0.06 Re_D^{2/3}\right) Pr^{0.4} \left(\frac{\mu}{\mu_s}\right)^{1/4}$$

where D is the diameter of the sphere and the Reynolds number, Re_D, is:.

$$Re_D = \frac{\rho u D}{\mu} = \frac{uD}{\nu}$$

Note that the fluid properties are evaluated at the fluid temperature, T_∞, except the viscosity, μ_s, evaluated at the surface temperature, T_s.

External Natural Convection over a Sphere

Math Model

Equation (9.35), page 585 in [BLID11]

$$\overline{Nu}_D = 2 + \frac{0.589 Ra_D^{1/4}}{\left[1 + (0.469/Pr)^{9/16}\right]^{4/9}}$$

where D is the diameter of the sphere and the the Rayleigh number, Ra_D, is:.

$$Ra_D = GrPr = \frac{g\rho^2 c\beta D^3 (T_s - T_\infty)}{k\mu} = \frac{g\beta D^3 (T_s - T_\infty)}{\nu\alpha}$$

Note that the fluid properties are evaluated at the film temperature, T_f:

$$T_f = \frac{T_s + T_\infty}{2}$$

Surface Radiation

Math Model

Radiation exchange between a surface and *large* surroundings
The heat flow rate is (Equation (1.7), p. 10 in [BLID11]):

$$Q = \sigma \epsilon_s A_s (T_s^4 - T_{sur}^4)$$

where σ is the Stefan-Boltzmann constant, ϵ_s is the surface emissivity and A_s is the area of the surface.
Note that the surface area, A_s, must be *much* smaller than the surrounding surface area, A_{sur}:

$$A_s \ll A_{sur}$$

Note that the temperatures must be the absolute temperature, K or $^\circ R$

Radiation Heat Transfer Coefficient

Math Model

Define the radiation heat transfer coefficient, h_r (see Equation (1.9), page 10 in [BLID11]):

$$h_r = \epsilon\sigma(T_s + T_{sur})(T_s^2 + T_{sur}^2)$$

Then,

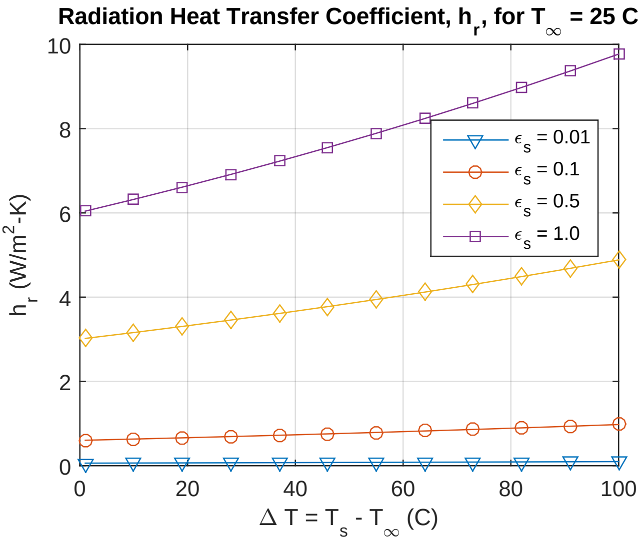
$$Q = h_r A_s(T_s - T_{sur})$$

Note:

- h_r is temperature dependent
- h_r can be used to compare the radiation to the convection heat transfer from a surface, h (if T_{sur} and T_∞ have similar values)

Range of Radiation Heat Transfer Coefficient

Math Model



TNSolver Input File

- What do we need in the input file for the lumped mass heat transfer experiment?
- Transient convection problem, with surface radiation
- Lumped capacitance approximation, $Bi < 0.1$, so no conduction in the solid object

Solution Parameters

TNSolver Input File

```
Begin Solution Parameters

  title = Lumped Mass Heat Transfer Experiment
  type = transient
  begin time   = (R)
  end time     = (R)
  time step    = (R)
  number of time steps = (I)

End Solution Parameters
```

(R) is a single real number
(I) is a single integer number

Nodes

TNSolver Input File

Define nodes which have a volume

```
Begin Nodes

! label    rho*c    V
   (S)      (R)    (R)

End Nodes
```

(S) is a single character string

Convection Conductor

TNSolver Input File

$$Q_{ij} = hA(T_s - T_\infty)$$

The heat transfer coefficient h is known.

```
Begin Conductors

! label    type      nd_i nd_j  parameters
    (S)   convection (S)   (S)   (R)   (R)   ! h, A

End Conductors
```

External Forced Convection (EFC) Conductor

TNSolver Input File

$$Q_{ij} = hA(T_s - T_\infty)$$

Heat transfer coefficient, h, is evaluated using the correlation for external forced convection from a sphere with diameter D and fluid velocity of u.

```
Begin Conductors
!                    Ts   Tinf
! label    type    nd_i nd_j parameters
  (S)   EFCsphere (S)   (S)   (S) (R) (R) ! material, u, D

End Conductors
```

Note that Re, Nu and h are reported in the output file.

External Natural Convection (ENC) Conductor

Thermal Network Model

$$Q_{ij} = hA(T_s - T_\infty)$$

Heat transfer coefficient, h, is evaluated using the correlation for external natural convection from a sphere with diameter *D*.

```
Begin Conductors
!                   Ts   Tinf
! label   type      nd_i nd_j  parameters
   (S)   ENCsphere  (S)  (S)    (S)   (R)   ! material, D

End Conductors
```

Note that Ra, Nu and h are reported in the output file.

Surface Radiation Conductor

TNSolver Input File

$$Q_{ij} = \sigma \epsilon A_s (T_s^4 - T_{env}^4)$$

σ is the Stefan-Boltzmann constant and ϵ is the surface emissivity.

```
Begin Conductors

! label type     nd_i nd_j parameters
   (S) surfrad  (S)  (S)  (R)  (R)  ! emissivity, A

End Conductors
```

Note that h_r is reported in the output file.

Boundary Conditions

TNSolver Input File

Specify a fixed temperature boundary condition, T_b, to one or more nodes in the model.

```
Begin Boundary Conditions

!  type        Tb       Node(s)
  fixed_T      (R)      (S ...)

End Boundary Conditions
```

(S ...) one or more character strings

Initial Conditions

TNSolver Input File

Specify the initial temperatures, T_0, to the nodes in the model.

```
Begin Initial Conditions

! T0   Node(s)
  (R)  (S ...)

End Initial Conditions
```

Example Input File

TNSolver Input File

```
Begin Solution Parameters
  title = Lumped Capacitance Experiment - Object A
  type = transient
  begin time  = 0.0
  end time    = 341.5
  time step   = 0.5
!  number of time steps = 20
End Solution Parameters

Begin Nodes
  1  3925000.0  6.2892e-05  !  rho*c,  V
End Nodes

Begin Conductors
  conv convection 1 Tinf 12.0 0.0076  ! h, A
! conv EFCsphere  1 Tinf air  13.13 0.04934  ! material, u, D
! conv ENCsphere  1 Tinf air  0.04934 ! material, D
  rad  surfrad    1 Tinf 0.95 0.0076  ! emissivity, A
End Conductors
```

Example Input File (continued)

TNSolver Input File

```
Begin Boundary Conditions
  fixed_T  25.0  Tinf
End Boundary Conditions

Begin Initial Conditions
  99.0   1  !  Ti, node
End Initial Conditions
```

TNSolver Output File Extensions

TNSolver Output Files

`.out`	ASCI text output file
`_node.csv`	spreadsheet CSV node data
`_cond.csv`	spreadsheet CSV conductor data file
`_timedata.csv`	spreadsheet CSV transient data file

Verification using Analytical Solution

TNSolver Verification

Backward Euler time integration is used in TNSolver.
How does time step affect accuracy?
Utilitize the analytical solution Equation (5.6), p. 282 in [BLID11]:

$$\frac{T - T_\infty}{T_i - T_\infty} = \exp\left[-\left(\frac{hA}{\rho c V}\right) t\right]$$

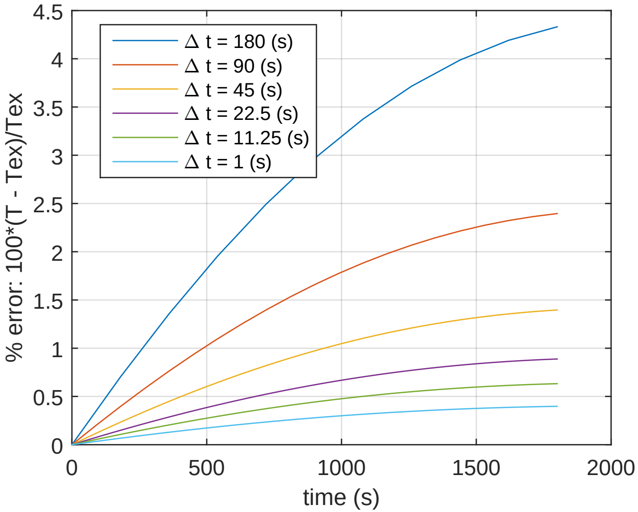
This is provided in the MATLAB function `lumpedmass.m`:

```
[T, Bi] = lumpedmass(time, rho, c, V, h, A, Ti, Tinf, k)
```

Example calculation using:
$D = 0.04931\ m$, $T_i = 100\ C$, $T_\infty = 25\ C$
$\rho = 7850\ kg/m^3$, $c = 500\ J/kg \cdot K$
$h = 25.0\ W/m^2 \cdot K$, $k = 62.0\ W/m \cdot K$

Verification using Analytical Solution

TNSolver Verification



Three MATLAB functions are provided for a least-squares analysis using TNSolver.

Recommend placing the experimental data into a MATLAB `.mat` file using `save` in order to `load` the experimental specimen temperature `expT`.

1. Estimate convection heat transfer coefficient, *h*, for the natural convection data using `ls_lumped_h.m`
2. Estimate velocity, *u*, for the forced convection data using `ls_lumped_vel.m`
3. Estimate surface emissivity, ϵ, using `ls_lumped_emiss.m`

Estimate h

Results

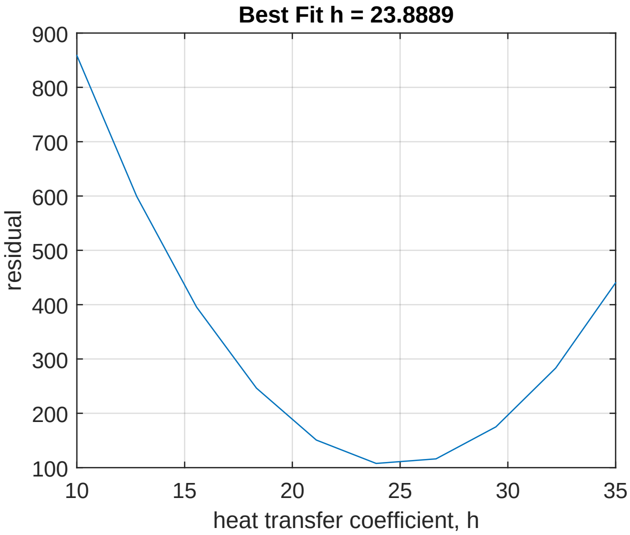
Example for object A, natural convection input file `ANC.inp`

1. Set begin and end time to match experimental data range
2. Set the time step to match experimental data sample rate
3. Set material properties and object geometries in input file
4. Set the boundary and initial conditions to match experiment
5. Use the `convection` conductor

```
>> load NC_A
>> h = linspace(10,35,10);
>> [besth] = ls_lumped_h('ANC', expT, h)
```

Estimate *h* Results and Plot

Results



Estimate Velocity

Results

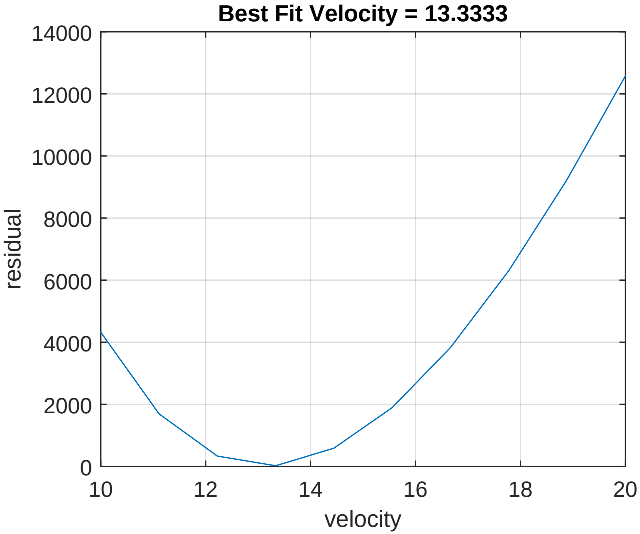
Example for object A, forced convection input file `AFC.inp`

1. Set begin and end time to match experimental data range
2. Set the time step to match experimental data sample rate
3. Set material properties and object geometries in input file
4. Set the boundary and initial conditions to match experiment
5. Use the `EFCsphere` convection conductor

```
>> load FC_A
>> u = linspace(10,20,10);
>> [bestvel] = ls_lumped_vel('AFC', expT, u)
```

Estimate Velocity Results and Plot

Results



Estimate Velocity

Results

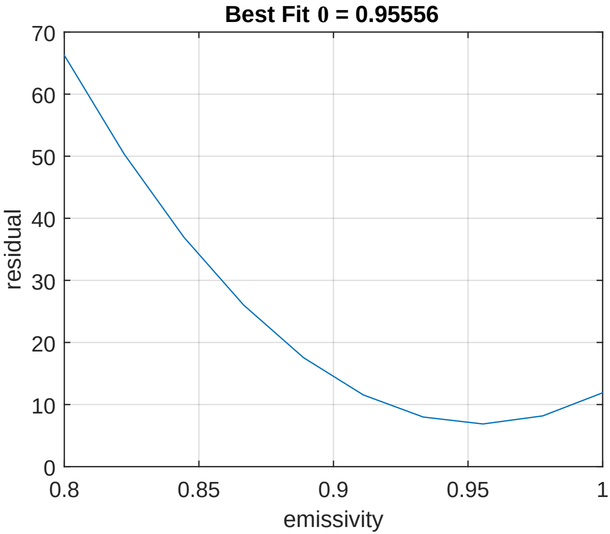
Example for object A, forced convection input file `AFC.inp`

1. Set begin and end time to match experimental data range
2. Set the time step to match experimental data sample rate
3. Set material properties and object geometries in input file
4. Set the boundary and initial conditions to match experiment
5. Use the `EFCsphere` convection conductor and the estimated velocity from previous

```
>> load FC_A
>> eps = linspace(.8,1,10);
>> [beste] = ls_lumped_emiss('AFC', expT, eps)
```

Estimate Emissivity Results and Plot

Results



Conclusion

- Math model for lumped capacitance method
- TNSolver input file described
- TNSolver thermal network model verification with analytical solution demonstrated
- Experimental data analysis

Questions?

References I

[BLID11] T.L. Bergman, A.S. Lavine, F.P. Incropera, and D.P. DeWitt.
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