Convection Heat Transfer Experiment

Thermal Network Solution with TNSolver

Bob Cochran
Applied Computational Heat Transfer
Seattle, WA
`rjc@heattransfer.org`

ME 331 Introduction to Heat Transfer
University of Washington
October 28, 2014
Expanded Revised Version: October 30, 2014

Outline

- Math Model
- Geometry
- Test Data
- Thermal Network Model Solution with TNSolver
- Results

What Is New, By Revision Date

October 30, 2014

1. Added geometry for 6.22” round copper fin
2. Added geometry for 11.22” square copper fin
3. Added network diagram for 6.22” fin model
4. Added notes for running the supplied model files

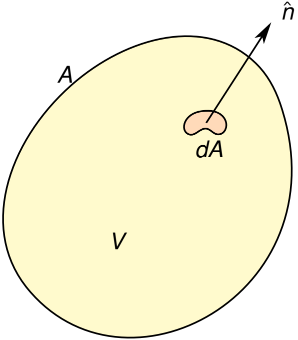
The Control Volume Concept

Math Model

$$\sum \text{Energy In} \quad \sum \text{Energy Out} =$$

$$\text{Energy Stored, Generated and/or Consumed}$$

Heat (transfer) is thermal energy transfer due to a temperature difference



Convection Correlations

Math Model

The heat flow rate is:

$$Q = hA(T_s \quad T_\infty)$$

where h is the heat transfer coefficient, T_s is the surface temperature and T_∞ is the fluid temperature

External Forced Convection over a Cylinder

Math Model

Equation (7.44), page 436 in [BLID11]

$$\overline{Nu}_D \qquad \frac{\bar{h}D}{k} = CRe_D^m Pr^{1/3}$$

where D is the diameter of the cylinder and the Reynolds number is $Re_D = \rho VD/\mu = VD/\nu$.

Re_D	C	m
0.4–4	0.989	0.330
4–40	0.911	0.385
40–4,000	0.683	0.466
4,000–40,000	0.193	0.618
40,000–400,000	0.027	0.805

Table 7.2, page 437 in [BLID11]

External Forced Convection over a Noncircular Cylinder

Math Model

For the case of a gas flowing over noncircular cylinders in crossflow (Table 7.3, page 437 in [BLID11]):

Geometry	Re_D	C	m
◆ $D = \sqrt{2H^2}$	6,000–60,000	0.304	0.59
■ $D = H$	5,000–60,000	0.158	0.66

Note that the fluid properties are evaluated at the film temperature, T_f:

$$T_f = \frac{T_s + T_\infty}{2}$$

External Natural Convection over a Horizontal Cylinder

Math Model

Equation (9.34), page 581 in [BLID11]:

$$\overline{Nu}_D = \frac{\bar{h}D}{k} = \left\{ 0.60 + \frac{0.387 Ra_D^{1/6}}{\left[1 + (0.559/Pr)^{9/16}\right]^{8/27}} \right\}^2$$

valid for $Ra_D \lesssim 10^{12}$, $Pr \gtrsim 0.7$, where D is the diameter of the cylinder and the Rayleigh number, Ra, is:

$$Ra_D = GrPr = \frac{g\rho^2 c\beta D^3 (T_s - T_\infty)}{k\mu} = \frac{g\beta D^3 (T_s - T_\infty)}{\nu\alpha}$$

Note that the fluid properties are evaluated at the film temperature, T_f:

$$T_f = \frac{T_s + T_\infty}{2}$$

Surface Radiation

Math Model

Radiation exchange between a surface and *large* surroundings
The heat flow rate is (Equation (1.7), page 10 in [BLID11]):

$$Q = \sigma \epsilon_s A_s (T_s^4 \quad T_{sur}^4)$$

where σ is the Stefan-Boltzmann constant, ϵ_s is the surface emissivity and A_s is the area of the surface.
Note that the surface area, A_s, must be *much* smaller than the surrounding surface area, A_{sur}:

$$A_s \quad A_{sur}$$

Note that the temperatures must be the absolute temperature, K or $^\circ R$

Radiation Heat Transfer Coefficient

Math Model

Define the radiation heat transfer coefficient, h_r (see Equation (1.9), page 10 in [BLID11]):

$$h_r = \epsilon\sigma(T_s + T_{sur})(T_s^2 + T_{sur}^2)$$

Then,

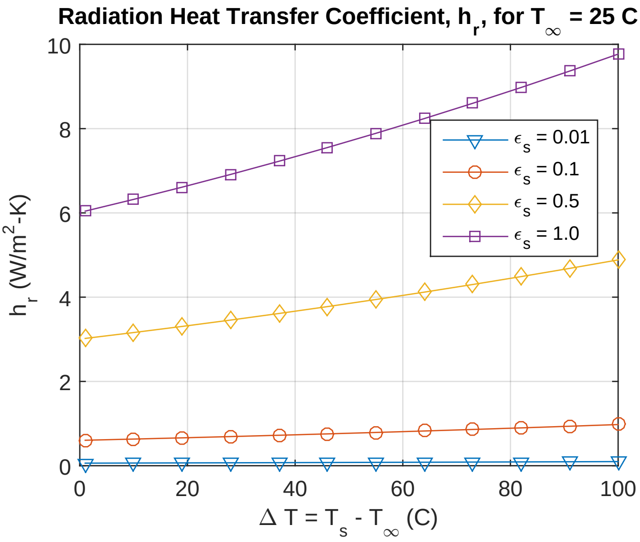
$$Q = h_r A_s(T_s - T_{sur})$$

Note:

- h_r is temperature dependent
- h_r can be used to compare the radiation to the convection heat transfer from a surface, h (if T_{sur} and T_∞ have similar values)

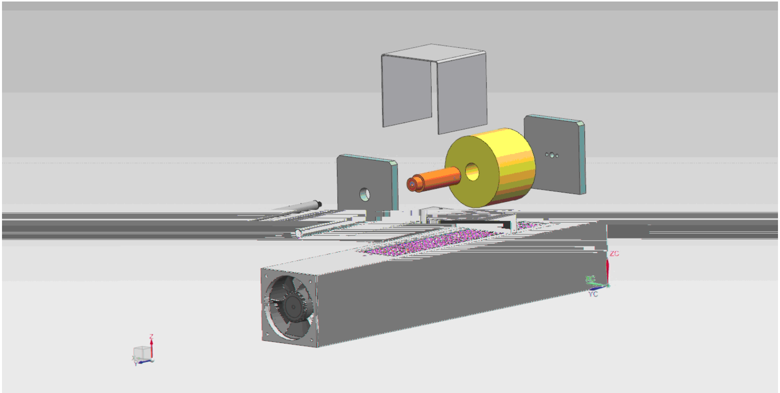
Range of Radiation Heat Transfer Coefficient

Math Model



Fin Convection Experiment

Geometry



Fins to be Studied

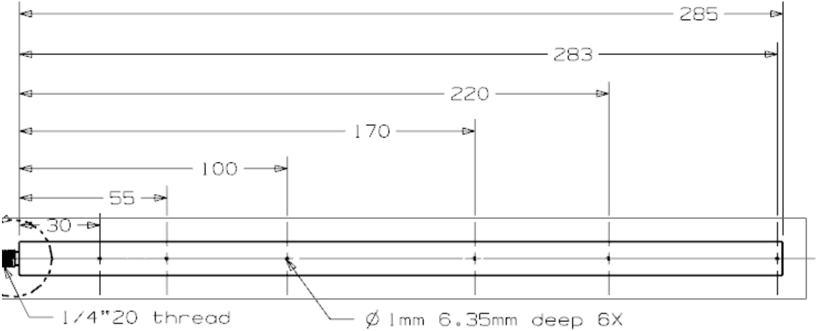
Geometry

- Four uniform cross section fins will be studied
- Three are circular, ●, and one is diamond shaped ◆ (a square, rotated 45°)

Fin Material	Dimensions	k (W/m-K)
Aluminum Alloy 6061-T6	0.5" ⌀ X 11.22" L	167.0
Copper Alloy 110	0.5" □ X 11.22" L	388.0
Copper Alloy 110	0.5" ⌀ X 6.77" L	388.0
Stainless Steel Alloy	0.5" ⌀ X 11.22" L	16.0

Thermocouple Locations

Geometry



Position Along the 0.285 m Fin (m)						
Base	TC1	TC2	TC3	TC4	TC5	TC6
0.0	.03	.055	.1	.170	.220	.283

Surface Radiation Properties

Geometry

Fin Material	Emissivity, ϵ (dimensionless)
Aluminum Alloy 6061-T6	0.09
Copper Alloy 110	0.12-0.15
Stainless Steel Alloy	0.5-0.7

See Table A.8, page 922, in [BLID11]

See Table 10.1, page 530, in [LL12]

Measurements

Test Data

The following measurements are made:

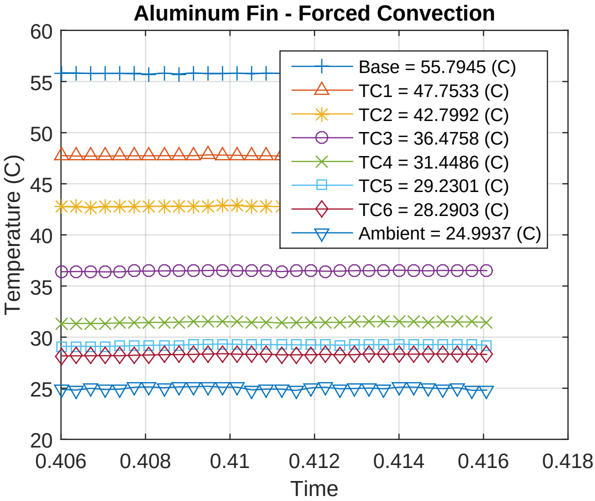
- Base temperature of the fin
- Heater power
- Ambient air temperature
- Fin temperatures along the length of the fin
  - Thermocouples are located at the center line of the fin
- Flow velocity for forced convection

A typical data set is shown for the aluminum fin, for both forced and natural convection.

Aluminum Fin - Forced Convection

Test Data

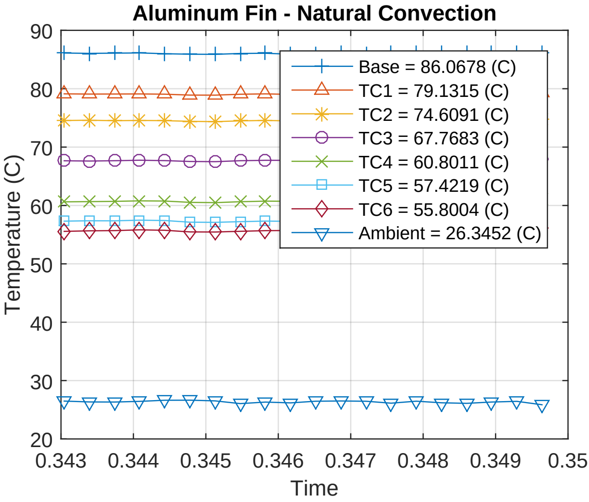
Heater Power = 10.53 W



Aluminum Fin - Natural Convection

Test Data

Heater Power = 10.53 W



Heat Transfer Analysis

Thermal Network Model

- Energy conservation: control volumes
- Identify and sketch out the control volumes
- Use the conductor analogy to represent energy transfer between the control volumes and energy generation or storage
  - Conduction, convection, radiation, other?
  - Capacitance
  - Sources or sinks
- State assumptions and determine appropriate parameters for each conductor
  - Geometry, material properties, etc.
- Which conductor(s)/source(s)/capacitance(s) are important to the required results?
  - Sensitivity analysis
- What is missing from the model? - peer/expert review

Thermal Network Terminology

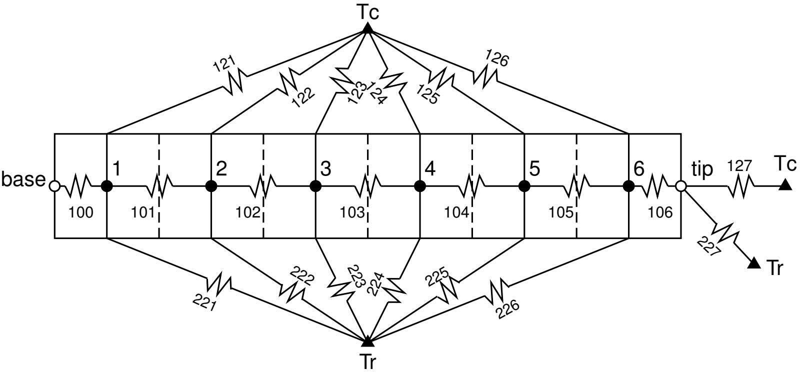
Thermal Network Model

- Geometry
  - Control Volume
    - Volume property, $V = \int_V dV$
    - Node: ●, $T_{\text{node}} = \int_V T(x_i)dV$
  - Control Volume Surface
    - Area property, $A = \int_A dA$
    - Surface Node: ○, $T_{\text{surface node}} = \int_A T(x_i)dA$
- Material properties
- Conductors
  - Conduction, convection, radiation
- Boundary conditions
  - Boundary node: ▲

Thermal Network Analysis

Thermal Network Model

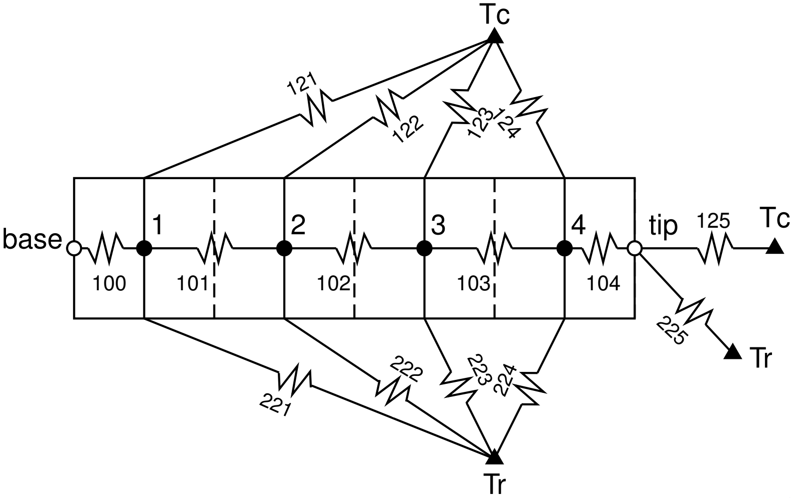
For the 11.22” long fin, there are six thermocouples:



Thermal Network Analysis

Thermal Network Model

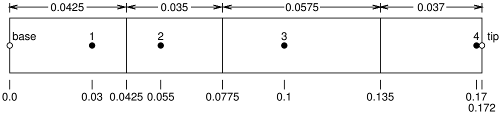
For the 6.77” long fin, there are four thermocouples:



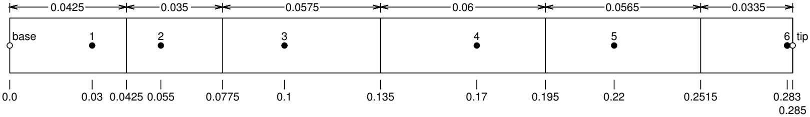
Control Volume Geometry

Thermal Network Model

Scale drawing of thermocouple locations and control volume lengths:



6.77” Fin Dimensions (m)



11.22” Fin Dimensions (m)

Note that the control volume boundaries are located midway between the thermocouple locations

Round Fin, 11.22” Long, Conduction Geometry

Thermal Network Model

Axial conduction in the fin, L = 11.22” = 0.285 m
D = 0.5” = 0.0127 m and r = 0.25” = 0.00635 m:

$L =$ distance between TC $\qquad A = \pi r^2$

Conductor	Length (m)	A (m^2)
100	0.030	0.00012667
101	0.025	0.00012667
102	0.045	0.00012667
103	0.070	0.00012667
104	0.050	0.00012667
105	0.063	0.00012667
106	0.002	0.00012667

Round Fin, 11.22” Long, Convection Geometry

Thermal Network Model

Surface convection area:

$$A = L_{CV}P = L_{CV}\pi D$$

D = 0.5” = 0.0127 m

Conductor	L_{CV} (m)	A (m^2)
121	0.0425	0.001696
122	0.0350	0.001396
123	0.0575	0.002294
124	0.0600	0.002394
125	0.0565	0.002254
126	0.0335	0.001337

Round Fin, 6.77” Long, Conduction Geometry

Thermal Network Model

Axial conduction in the fin, L = 6.77” = 0.172 m
D = 0.5” = 0.0127 m and r = 0.25” = 0.00635 m:

$L =$ distance between TC $\qquad A = \pi r^2$

Conductor	Length (m)	A (m^2)
100	0.030	0.00012667
101	0.025	0.00012667
102	0.045	0.00012667
103	0.070	0.00012667
104	0.002	0.00012667

Round Fin, 6.77” Long, Convection Geometry

Thermal Network Model

Surface convection area:

$$A = L_{CV} P = L_{CV} \pi D$$

D = 0.5” = 0.0127 m

Conductor	L_{CV} (m)	A (m^2)
121	0.0425	0.001696
122	0.0350	0.001396
123	0.0575	0.002294
124	0.0370	0.001476

Square Fin, 11.22” Long, Conduction Geometry

Thermal Network Model

Axial conduction in the fin, L = 11.22” = 0.285 m, H = 0.5” = 0.0127 m

$$L = \text{distance between TC} \qquad A = H^2$$

Conductor	Length (m)	A (m^2)
100	0.030	0.000016129
101	0.025	0.000016129
102	0.045	0.000016129
103	0.070	0.000016129
104	0.050	0.000016129
105	0.063	0.000016129
106	0.002	0.000016129

Square Fin, 11.22” Long, Convection Geometry

Thermal Network Model

Surface convection area:

$$A = L_{CV} P = L_{CV} 4H$$

H = 0.5” = 0.0127 m

Conductor	L_{CV} (m)	A (m^2)
121	0.0425	0.002159
122	0.0350	0.001778
123	0.0575	0.002921
124	0.0600	0.003048
125	0.0565	0.0028702
126	0.0335	0.0017018

Conduction Conductor

Thermal Network Model

$$Q_{ij} = \frac{kA}{L}(T_i \quad T_j)$$

```
Begin Conductors

! label type        nd_i nd_j    k     L         A
  100   conduction  base   1    x.x  0.030  0.00012667
  101   conduction   1     2    x.x  0.025  0.00012667
  102   conduction   2     3    x.x  0.045  0.00012667
  103   conduction   3     4    x.x  0.070  0.00012667
  104   conduction   4     5    x.x  0.050  0.00012667
  105   conduction   5     6    x.x  0.063  0.00012667
  106   conduction   6    tip   x.x  0.002  0.00012667

End Conductors
```

Convection Conductor

Thermal Network Model

$$Q_{ij} = hA(T_s \quad T_\infty) = hA(T_i \quad T_j)$$

```
Begin Conductors

! label    type      nd_i nd_j  h       A
    121   convection   1    Tc   x.x   0.001696
    122   convection   2    Tc   x.x   0.001396
    123   convection   3    Tc   x.x   0.002294
    124   convection   4    Tc   x.x   0.002394
    125   convection   5    Tc   x.x   0.002254
    126   convection   6    Tc   x.x   0.001337
    127   convection tip    Tc   x.x   0.00012667

End Conductors
```

Forced Convection Over Diamond Conductor

Thermal Network Model

Heat transfer coefficient, h, is evaluated using the correlation.

```
Begin Conductors

! label type       nd_i   nd_j fluid  V     D         A
  121   EFCdiamond   1     Tc    air  x.x  0.018    0.002159
  122   EFCdiamond   2     Tc    air  x.x  0.018    0.001778
  123   EFCdiamond   3     Tc    air  x.x  0.018    0.002921
  124   EFCdiamond   4     Tc    air  x.x  0.018    0.003048
  125   EFCdiamond   5     Tc    air  x.x  0.018    0.0028702
  126   EFCdiamond   6     Tc    air  x.x  0.018    0.0017018
  127   EFCdiamond  tip    Tc    air  x.x  0.0127   0.00012667

End Conductors
```

Note that Re, Nu and h are reported in the output file.

Natural Convection Conductor

Thermal Network Model

Heat transfer coefficient, h, is evaluated using the correlation.

```
Begin Conductors

! label type   nd_i   nd_j fluid  D        A
  121   ENChcyl  1     Tc    air  0.0127   0.001696
  122   ENChcyl  2     Tc    air  0.0127   0.001396
  123   ENChcyl  3     Tc    air  0.0127   0.002294
  124   ENChcyl  4     Tc    air  0.0127   0.002394
  125   ENChcyl  5     Tc    air  0.0127   0.002254
  126   ENChcyl  6     Tc    air  0.0127   0.001337
  127   ENChcyl tip    Tc    air  0.0127   0.00012667

End Conductors
```

Note that Ra, Nu and h are reported in the output file.

Surface Radiation Conductor

Thermal Network Model

$$Q_{ij} = \sigma \epsilon_i A_i (T_i^4 \quad T_j^4)$$

```
Begin Conductors

! label type    nd_i nd_j emissivity     A
  221   surfrad  1    Tr    x.xx      0.001696
  222   surfrad  2    Tr    x.xx      0.001396
  223   surfrad  3    Tr    x.xx      0.002294
  224   surfrad  4    Tr    x.xx      0.002394
  225   surfrad  5    Tr    x.xx      0.002254
  226   surfrad  6    Tr    x.xx      0.001337
  227   surfrad tip   Tr    x.xx      0.00012667

End Conductors
```

Note that h_r is reported in the output file.

Boundary Conditions

Thermal Network Model

```
Begin Boundary Conditions

!  type        Tb      Node(s)
  fixed_T     x.xx      Tc    ! fluid T
  fixed_T     x.xx      Tr    ! surrounding radiation T
  fixed_T     x.xx      base  ! fin base T

End Boundary Conditions
```

TNSolver Input Files

Thermal Network Model

The supplied TNSolver input files for each fin model:

Fin	Forced Convection	Natural Convection
Aluminum, 11.22”	`Al_fin_FC.inp`	`Al_fin_NC.inp`
Round Copper, 6.77”	`Cu_fin_FC.inp`	`Cu_fin_NC.inp`
Square Copper, 11.22”	`sqCu_fin_FC.inp`	`sqCu_fin_NC.inp`
Stainless Steel, 11.22”	`SS_fin_FC.inp`	`SS_fin_NC.inp`

Edit each file to set appropriate boundary conditions (Tc, Tr and base) and flow velocities (forced convection models) for your specific experimental data set

Running a TNSolver Model

Thermal Network Model

The MATLAB/Octave command to run the aluminum fin, forced convection model is:

```
>>tnsolver('Al_fin_FC');
```

Note that you supply the base of the input file name (`AL_fin_FC`, in this example), without the `.inp` extension

The results output file will be the base name with the extension `.out` (`Al_fin_FC.out`, in this example)

Screen Output When Running TNSolver

Thermal Network Model

```
**********************************************************
*                                                        *
*          TNSolver - A Thermal Network Solver           *
*                                                        *
*            Version 0.2.0, October 28, 2014             *
*                                                        *
**********************************************************

Reading the input file: Al_fin_FC.inp

Initializing the thermal network model ...

Starting solution of a steady thermal network model ...

     Nonlinear Solve
  Iteration    Residual
  ---------  ------------
        1     52.9978
        2     0.0309635
        3     1.21353e-05
        4     5.09316e-09

Results have been written to: Al_fin_FC.out

All done ...
```

View TNSolver Output File For Results

Thermal Network Model

Open `Al_fin_FC.out` in your favorite text editor

```
**********************************************************
*                                                        *
*          TNSolver - A Thermal Network Solver           *
*                                                        *
*            Version 0.2.0, October 28, 2014             *
*                                                        *
**********************************************************

Model run finished at 10:05 AM, on October 29, 2014

*** Solution Parameters ***

  Title: Aluminum Fin - Forced Convection

  Type                           =  steady
  Units                          =  SI
  Temperature units              =  C
  Nonlinear convergence          =  1e-008
  Maximum nonlinear iterations   =  15
  Gravity                        =  9.80665 (m/s^2)
  Stefan-Boltzmann constant      =  5.67037e-008 (W/m^2-K^4)
```

View TNSolver Output File For Results (continued)

Thermal Network Model

```
*** Nodes ***

                              Volume   Temperature
   Label     Material         (m^3)        (C)
 --------- ---------- ---------- -----------
         1        N/A          0     47.1038
         2        N/A          0      42.202
         3        N/A          0     36.0832
         4        N/A          0     31.0199
         5        N/A          0      29.207
         6        N/A          0     28.4181
        Tc        N/A          0          25
        Tr        N/A          0          25
      base        N/A          0        55.8
       tip        N/A          0      28.416
```

View TNSolver Output File For Results (continued)

Thermal Network Model

```
*** Conductors ***

                                             Q_ij
    Label      Type    Node i    Node j       (W)
 ---------- ---------- ---------- ---------- ----------
        100 conduction       base          1    6.13192
        101 conduction          1          2    4.14771
        102 conduction          2          3    2.87638
        103 conduction          3          4    1.53011
        104 conduction          4          5   0.766993
        105 conduction          5          6   0.264889
        106 conduction          6        tip  0.0229109
        121     EFCcyl          1         Tc    1.96157
        122     EFCcyl          2         Tc    1.25717
        123     EFCcyl          3         Tc    1.33172
        124     EFCcyl          4         Tc   0.755085
        125     EFCcyl          5         Tc   0.496864
        126     EFCcyl          6         Tc   0.239463
        127     EFCcyl        tip         Tc  0.0226728
        221    surfrad          1         Tr  0.0226511
        222    surfrad          2         Tr  0.0141605
        223    surfrad          3         Tr  0.0145417
        224    surfrad          4         Tr 0.00803642
        225    surfrad          5         Tr 0.00523996
        226    surfrad          6         Tr 0.00251536
        227    surfrad        tip         Tr 0.000238156
```

View TNSolver Output File For Results (continued)

Thermal Network Model

```
*** Conductor Parameters ***

surfrad: Surface Radiation

                  h_r
    label     (W/m^2-K)
 ---------- ----------
        221    0.60422
        222   0.589676
        223   0.571949
        224   0.557634
        225   0.552586
        226   0.550402
        227   0.550396

EFCcyl: External Forced Convection - Cylinder

                                            h
    label     Re Number  Nu Number  (W/m^2-K)
 ---------- ---------- ---------- ----------
        121    2832.69    24.7609     52.325
        122    2869.36    24.9192    52.3515
        123    2917.02    25.1207    52.3789
        124    2958.53    25.2909    52.3941
        125    2973.94    25.3526    52.3974
        126    2980.75    25.3797    52.3984
        127    2980.77    25.3798    52.3984
```

Verification with Analytical Solution

Results

Table 3.4, page 161 in [BLID11], Temperature distribution and heat loss for fins with a uniform cross section

Case	Tip Condition	Temperature, θ/θ_b	Fin Heat Transfer Rate, Q
A	Convection	$\frac{\cosh m(L-x)+(h/mk)\sinh m(L-x)}{\cosh mL+(h/mk)\sinh ml}$	$M\frac{\sinh mL+(h/mk)\cosh mL}{\cosh mL+(h/mk)\sinh mL}$
B	Adiabatic	$\frac{\cosh m(L-x)}{\cosh mL}$	$M\tanh mL$
C	Specified T	$\frac{(\theta_L/\theta_b)\sinh mx+\sinh m(L-x)}{\sinh mL}$	$M\frac{\cosh mL-(\theta_L/\theta_b)}{\sinh mL}$
D	$L = 1$	e^{mx}	M

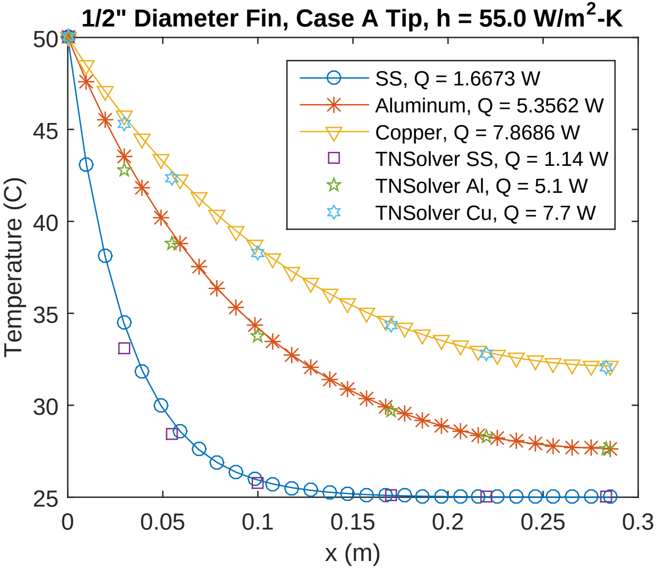
$\theta = T \quad T_\infty$ $\qquad m = \sqrt{hP/kA_c}$

$\theta_b = \theta(0) = T_b \quad T_\infty$ $\qquad M = (\sqrt{hPkA_c})\theta_b$

For circular fins: $P = \pi D$ and $A_c = \pi(D/2)^2$
For square/diamond fins: $P = 4H$ and $A_c = H^2$

Comparison with TNSolver Network Model

Results



TNSolver Forced Convection Correlation

Results

Aluminum fin, flow velocity = 3.7 m/s = 8.3 mph
$T_{amb} = 25.0C$ and $T_{base} = 55.8C$

TC	Experiment (C)	TNSolver (C)	Q_c (W)	Q_r (W)	h $(W/m^2 \quad K)$	h_r $(W/m^2 \quad K)$
1	47.8	47.1	1.96	0.022	52.3	0.60
2	42.8	42.2	1.26	0.014	52.3	0.59
3	36.5	36.1	1.33	0.015	52.3	0.57
4	31.4	31.0	0.75	0.008	52.4	0.56
5	29.2	29.3	0.50	0.005	52.4	0.55
6	28.4	28.5	0.24	0.003	52.4	0.55
tip		28.4	0.023	0.00024	52.4	0.55

Q_f = 6.1 W
Re range: 2,833 – 2,981
Nu range: 24.8 – 25.4

TNSolver Natural Convection Correlation

Results

Aluminum fin

$T_{amb} = 26.3$ and $T_{base} = 86.1 C$

TC	Experiment (C)	TNSolver (C)	Q_c (W)	Q_r (W)	h $(W/m^2 \quad K)$	h_r $(W/m^2 \quad K)$
1	79.1	82.2	0.421	0.0683	4.4	0.72
2	74.6	79.4	0.327	0.0528	4.4	0.71
3	67.8	75.4	0.491	0.0786	4.3	0.70
4	60.8	70.9	0.460	0.0729	4.3	0.68
5	57.4	69.0	0.412	0.0651	4.2	0.68
6	55.8	68.0	0.238	0.0375	4.2	0.67
tip		67.9	0.0226	0.00359	4.2	0.67

Q_f = 2.8 W

Ra range: 6,000 – 7,300

Nu range: 1.96 – 2.01

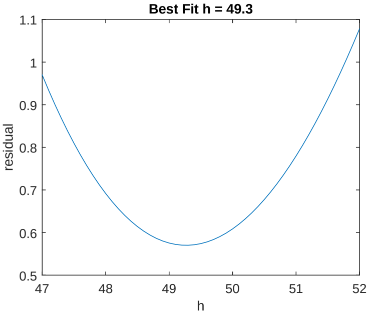
Al Fin Forced Convection Least Squares Fit h

Results

A constant heat transfer coefficient, *h*, was determined using a least squares residual between the experimental data and the TNSolver thermal network model.

$$h = 49.3 \; W/m^2 \quad K$$

TC	Experiment T (C)	TNSolver T (C)
1	47.8	47.3
2	42.8	42.5
3	36.5	36.5
4	31.5	31.4
5	29.2	29.5
6	28.3	28.7



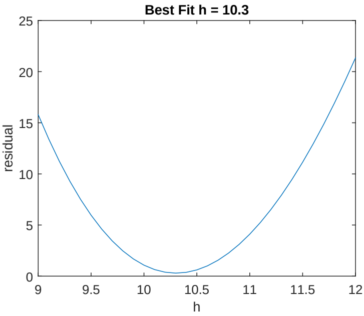
Al Fin Natural Convection Least Squares Fit h

Results

A constant heat transfer coefficient, *h*, was determined using a least squares residual between the experimental data and the TNSolver thermal network model.

h = 10.3 $W/m^2 \quad K$

TC	Experiment T (C)	TNSolver T (C)
1	79.1	79.0
2	74.6	74.4
3	67.8	67.6
4	60.8	60.5
5	57.4	57.6
6	55.8	56.1



Conclusion

- TNSolver thermal network model verification with analytical solution demonstrated validity of the model
- The forced convection correlation was within experimental error
- The natural convection correlation was within 20% of the experimental temperature data
- The forced convection heat transfer coefficient determined using least squares fitting was within 10% of the correlation
- The natural convection heat transfer coefficient determined using least squares fitting was 2.3 times the correlation h

Questions?

Text Editors

The input file to TNSolver is a plain text file

It is not a good idea to use a word processor to edit the files

Some recommendations:

- Cross-platform: vim/gvim, emacs, Bluefish, among many others
- Windows: notepad, Notepad++
- MacOS: TextEdit, Smultron
- Linux: see cross-platform options

References I

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