Thermal Network Analysis with TNSolver

Steady Conduction - The Composite Wall Problem

Bob Cochran
Applied Computational Heat Transfer
Seattle, WA
rjc@heattransfer.org

ME 331 Introduction to Heat Transfer
University of Washington
October 13, 2015

Outline

Heat Transfer Analysis using Thermal Networks

- Heat Transfer Math Model
  - Steady, plane wall conduction with convection
  - Control volumes and the integral form
- Introduction to TNSolver
- Composite Wall Problem

Conduction, Convection and Radiation

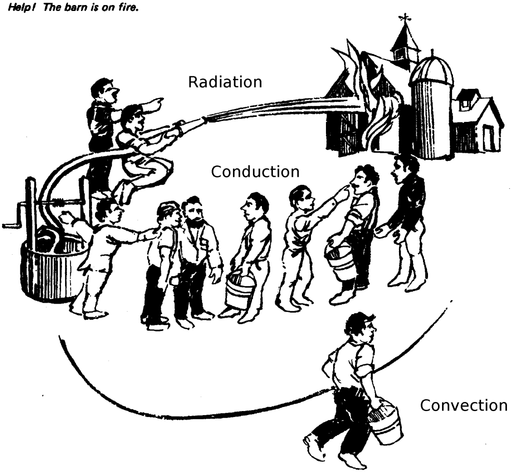
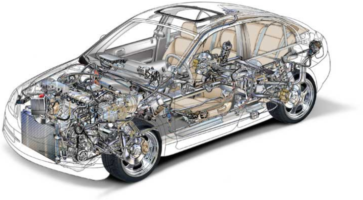
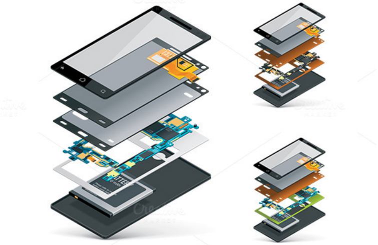


Figure borrowed from [LL12].

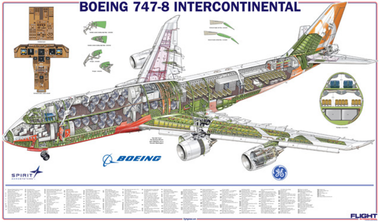
Automotive



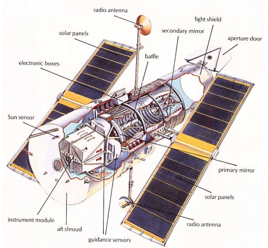
Electronics Packaging



Aircraft



Aerospace

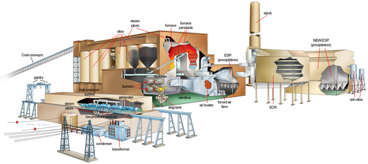


Heat Transfer in Industry

Math Model

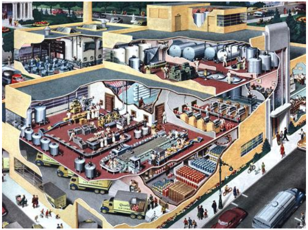
HVAC

Energy Production



Naval

Food Production



Heat Transfer Analysis

Math Model

Answering design questions about thermal energy and temperature

- Hand calculation - back-of-the-envelope
  - On the order of 1-10 equations
- Spreadsheet style
  - Interactive Heat Transfer (IHT 4.0), see p. ix in [BLID11]
  - LibreOffice Calc, Microsoft Excel, MathCAD
- Thermal network or lumped parameter approach
  - On the order of 10-1,000 equations
- Continuum approach - solid model/mesh generation
  - On the order of 1,000-1,000,000 equations
  - Finite Volume Method (FVM)
  - Finite Element Method (FEM)

See Section 1.5, page 38, in [BLID11]

Commercial Thermal Network Solvers

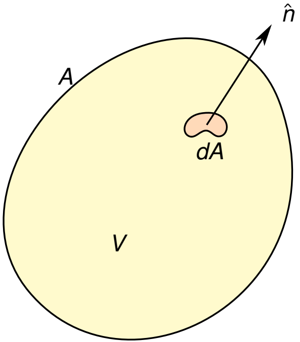
- C&R Technologies
  - SINDA/FLUINT, Thermal Desktop, RadCAD
- MSC Software
  - Sinda, SindaRad, Patran
- ESATAN-TMS
  - Thermal, Radiative, CADbench

The Control Volume Concept

Math Model

$$\sum \text{Energy In} - \sum \text{Energy Out} = \text{Energy Stored, Generated and/or Consumed}$$

Heat (transfer) is thermal energy transfer due to a temperature difference



Integral Form of Steady Heat Conduction

Math Model

The steady conduction equation, in Cartesian tensor integral form, is:

$$\int_A q_i n_i \, dA = \iint_V \dot{q} \, dV$$

where $\dot{q}$ is a volumetric source and Fourier's Law of Heat Conduction provides a constitutive model for the heat flux as a function of temperature gradient:

$$q_i = -k \frac{\partial T}{\partial x_i}$$

where k is the isotropic thermal conductivity.

Convection

Math Model

Convection heat transfer from the surface of the control volume is modeled by:

$$\int_{\Gamma_c} q_i n_i \, dA = \int_{\Gamma_c} h(T_s - T_c) \, dA, \quad \text{where} \begin{cases} T_s > T_c, & \text{cooling} \\ T_s < T_c, & \text{heating} \end{cases}$$

The convection coefficient, $h(x_i, t, T_s, T_c)$, is usually a function of position, time, surface temperature, T_s, free stream or bulk temperature, T_c, and other parameters. The value of the coefficient is often evaluated using a correlation.

Introducing TNSolver

- Thermal Network Solver - TNSolver
- MATLAB/Octave program
  - GNU Octave is an open source clone of MATLAB
- Thermal model is described in a text input file
  - Do not use a word processor, use a text editor, such as:
    - Cross-platform: vim/gvim, emacs, Bluefish, among many others
    - Windows: notepad, Notepad++
    - MacOS: TextEdit, Smultron
    - Linux: see cross-platform options
- Simulation results are both returned from the function and written to text output files for post-processing

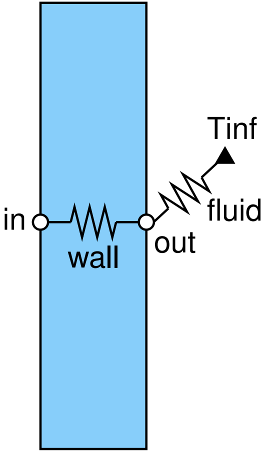
Thermal Network Terminology

- Time dependency
  - Steady state or transient
  - Initial condition is required for transient
- Geometry
  - Control Volume - volume, $V = \int_V dV$
    - Node: ●, $T_{\text{node}} = \int_V T(x_i) dV$
  - Control Volume Surface - area, $A = \int_A dA$
    - Surface Node: ○, $T_{\text{surface node}} = \int_A T(x_i) dA$
- Material properties
- Conductors
  - Conduction
  - Convection
  - Radiation
- Boundary conditions
  - Boundary node: ▲
- Sources/sinks

TNSolver Input Example of Text Input File

TNSolver User Guide

```
!  Simple Wall Model

Begin Solution Parameters
  type  = steady
End Solution Parameters

Begin Conductors
 wall  conduction in  out   2.3  1.2   1.0 ! k L A
 fluid convection out Tinf  2.3  1.0       ! h A
End Conductors

Begin Boundary Conditions
  fixed_T  21.0   in    ! Inner wall T
  fixed_T  5.0    Tinf  ! Fluid T
End Boundary Conditions
```



! begins a comment (MATLAB uses %)

Solution Parameters

TNSolver User Guide

```
Begin Solution Parameters

  title = A thermal network model
  type  = steady   ! <steady|transient>
  units = SI       ! <SI|US>

End Solution Parameters
```

Conduction: Cartesian (The Plane Wall)

The rate of heat transfer, Q_{ij}, due to conduction, between the two temperatures T_i and T_j, separated by a distance L and area A, is:

$$Q_{ij} = \frac{kA}{L}\left(T_i - T_j\right)$$

The heat flux, q_{ij}, is:

$$q_{ij} = \frac{Q_{ij}}{A} = \frac{k}{L}\left(T_i - T_j\right)$$

```
Begin Conductors

! label  type        node i  node j  parameters
  name  conduction   label   label   x.x x.x x.x  ! k L A

End Conductors
```

Convection Conductor

The rate of heat transfer due to convection is:

$$Q_{ij} = hA\left(T_i - T_j\right)$$

```
Begin Conductors

! label  type        node i node j  parameters
  name convection   label  label    x.x x.x    ! h A

End Conductors
```

Specified Surface Temperature Boundary Condition

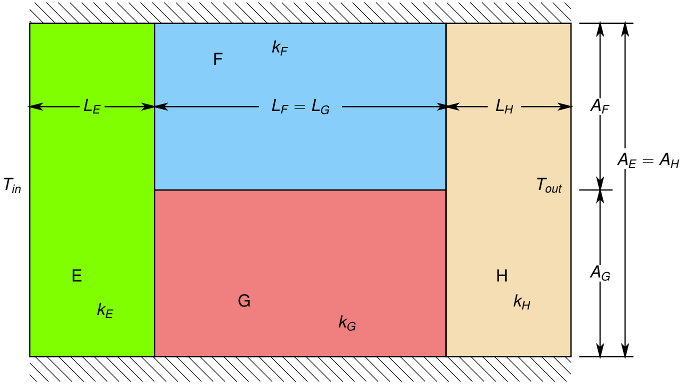
The node temperature, T_b, is specified:

```
Begin Boundary Conditions

! type     parameter(s)   node(s)
 fixed_T  T_b             label

End Boundary Conditions
```

Description of the Composite Wall Problem

Composite Wall Model

Consider a composite wall:



See Figure 3.3, on page 117, in [BLID11].

Model Parameters

Composite Wall Model

The inner wall temperature $T_{in} = 1$

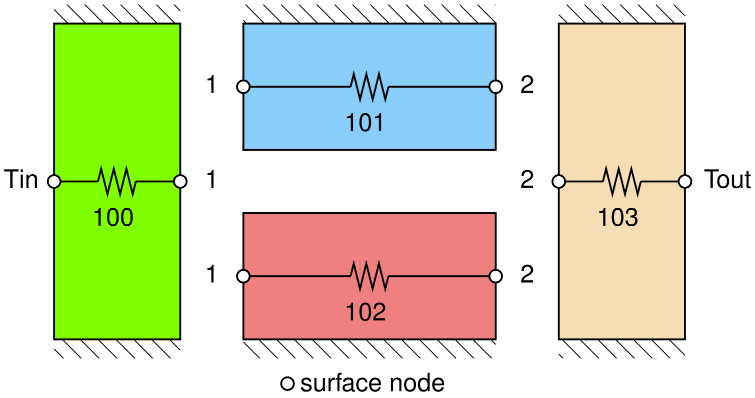
The outer wall temperature $T_{out} = 0$

Region	Conductivity, k	Length, L	Area, A
E	1.0	1.0	2.0
F	2.0	2.0	1.0
G	$0.001 \leq k_G \leq 2.0$	2.0	1.0
H	3.0	1.0	2.0

First Approach

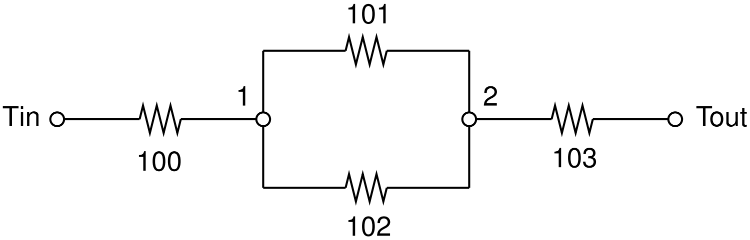
Composite Wall Model 1

There are four control volumes:



Network Diagram

Composite Wall Model 1



Compare with Figure 3.3 (a), on page 117, in [BLID11].

TNSolver Input File for $k_G = 2.0$

Composite Wall Model 1

```
!  Composite wall model: Approach 1 series-parallel

Begin Solution Parameters

  type   = steady

End Solution Parameters

Begin Conductors

! label  type        node 1   node 2   parameters
   100  conduction   Tin       1       1.0  1.0  2.0 ! k_E L_E A_E
   101  conduction   1         2       2.0  2.0  1.0 ! k_F L_F A_F
   102  conduction   1         2       2.0  2.0  1.0 ! k_G L_G A_G
   103  conduction   2        Tout     3.0  1.0  2.0 ! k_H L_H A_H

End Conductors

Begin Boundary Conditions

! type    parameter(s)   node(s)
 fixed_T     1.0          Tin     ! inner wall temperature
 fixed_T     0.0          Tout    ! outer wall temperature

End Boundary Conditions
```

TNSolver Output for $k_G = 2.0$

Composite Wall Model 1

Nodes

Label	Material	Volume (m^3)	Temperature (C)
Tin	N/A	0	1
1	N/A	0	0.571429
2	N/A	0	0.142857
Tout	N/A	0	0

Conductors

Label	Type	Node i	Node j	Q_ij (W)
100	conduction	Tin	1	0.857143
101	conduction	1	2	0.428571
102	conduction	1	2	0.428571
103	conduction	2	Tout	0.857143

Verification of the Results

Composite Wall Model 1

Analytical solution is used to compare to TNSolver results

Using Equation (3.19), page 116, in [BLID11]:

$$R_{tot} = \frac{1}{UA} = \frac{L_E}{k_E A_E} + \left[\frac{k_F A_F}{L_F} + \frac{k_G A_G}{L_G} \right]^{-1} + \frac{L_H}{k_H A_H}$$

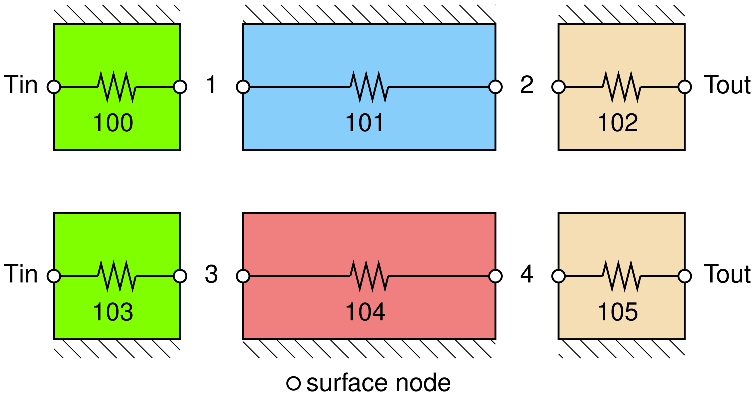
$$R_{tot} = \frac{1}{UA} = \frac{1}{(1)(2)} + \left[\frac{(2)(1)}{2} + \frac{(2)(1)}{2} \right]^{-1} + \frac{1}{(3)(2)} = \frac{7}{6} = 1.1667$$

$$Q = UA\Delta T = \frac{(T_{in} - T_{out})}{R_{tot}} = \frac{(1.0 - 0.0)}{1.1667} = 0.857$$

Second Approach

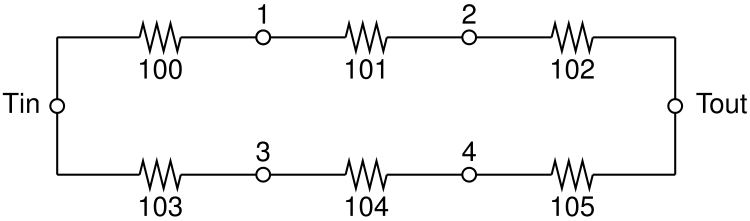
Composite Wall Model 2

There are six control volumes:



Network Diagram

Composite Wall Model 2



Compare with Figure 3.3 (b), on page 117, in [BLID11].

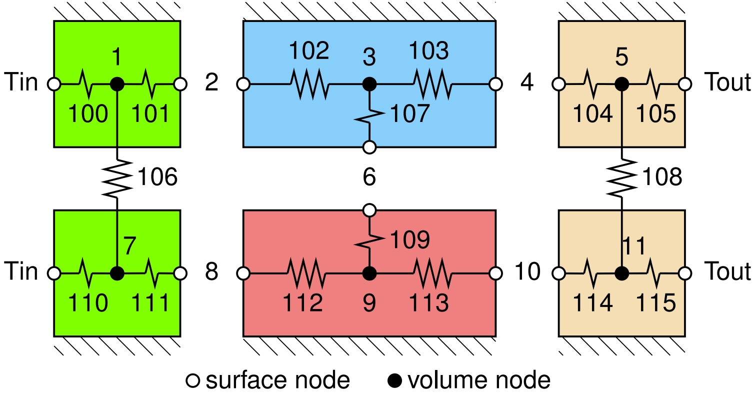
TNSolver Input File for $k_G = 2.0$

Composite Wall Model 2

```
!  Composite wall model: Approach 2 - parallel conductors

Begin Solution Parameters

  type  = steady

End Solution Parameters

Begin Conductors

! label  type        node 1  node 2  parameters
   100  conduction   Tin      1       1.0, 1.0, 1.0  ! k_E L_E A_E
   101  conduction   1        2       2.0, 2.0, 1.0  ! k_F L_F A_F
   102  conduction   2        Tout    3.0, 1.0, 1.0  ! k_H L_H A_H
   103  conduction   Tin      3       1.0, 1.0, 1.0  ! k_E L_E A_E
   104  conduction   3        4       2.0, 2.0, 1.0  ! k_G L_G A_G
   105  conduction   4        Tin     3.0, 1.0, 1.0  ! k_H L_H A_H

End Conductors

Begin Boundary Conditions

! type    parameter(s)   node(s)
 fixed_T      1.0          Tin    ! inner wall temperature
 fixed_T      0.0          Tout   ! outer wall temperature

End Boundary Conditions
```

Third Approach
Composite Wall Model 3

There are six control volumes:



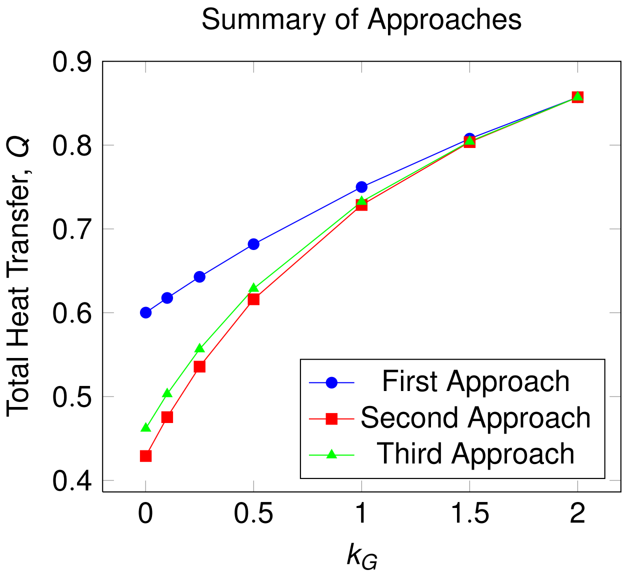
TNSolver Input File for $k_G = 2.0$

Composite Wall Model 3

```
!  Composite wall model: Approach 3

Begin Solution Parameters

  type  = steady

End Solution Parameters

Begin Conductors

! label  type       node 1  node 2  parameters
   100  conduction   Tin     1        1.0, 0.5, 1.0  ! k_E L_E A_E/2
   101  conduction   1       2        1.0, 0.5, 1.0  ! k_E L_E A_E/2
   102  conduction   2       3        2.0, 1.0, 1.0  ! k_F L_F A_F
   103  conduction   3       4        2.0, 1.0, 1.0  ! k_F L_F A_F
   104  conduction   4       5        3.0, 0.5, 1.0  ! k_H L_H A_H/2
   105  conduction   5      Tout      3.0, 0.5, 1.0  ! k_H L_H A_H/2

   106  conduction   1       7        1.0, 1.0, 1.0  ! k_E
   107  conduction   3       6        2.0, 0.5, 2.0  ! k_F
   108  conduction   5      11        3.0, 1.0, 1.0  ! k_H
   109  conduction   6       9        2.0, 0.5, 2.0  ! k_G

   110  conduction   Tin     7        1.0, 0.5, 1.0  ! k_E L_E A_E/2
   111  conduction   7       8        1.0, 0.5, 1.0  ! k_E L_E A_E/2
   112  conduction   8       9        2.0, 1.0, 1.0  ! k_G L_G A_G
   113  conduction   9      10        2.0, 1.0, 1.0  ! k_G L_G A_G
   114  conduction  10      11        3.0, 0.5, 1.0  ! k_H L_H A_H/2
   115  conduction  11      Tout      3.0, 0.5, 1.0  ! k_H L_H A_H/2

End Conductors

Begin Boundary Conditions

! type    parameter(s)   node(s)
 fixed_T      1.0          Tin    ! inner wall temperature
 fixed_T      0.0          Tout   ! outer wall temperature

End Boundary Conditions
```

Total Heat Transfer over the Range of k_G

Composite Wall Model Summary



Conclusion

- Heat Transfer Analysis in Industry
- Thermal Network Analysis Method
  - Open source TNSolver for Octave/MATLAB
  - Steady, Cartesian conduction and convection conductors
- The Composite Wall Problem
  - Three control volume approaches
  - Overall total heat flow, Q, for $0.001 \leq k_G \leq 2.0$

Questions?

Obtaining GNU Octave

- GNU Octave
  - http://www.gnu.org/software/octave/
- Octave Wiki
  - http://wiki.octave.org
- Octave-Forge Packages (similar to MATLAB Toolbox packages)
  - http://octave.sourceforge.net
- For Windows installation I would suggest the MinGW installation.
  - If you already have Cygwin installed, then install that version.

SI Units

Quantity	Symbol		Fundamental	Derivatives
Mass	m	M	kg	
Length	x, y, z	L	m	
Area	A	L^2	m^2	
Volume	V	L^3	m^3	
Time	t	t	s	
Force	F	$\frac{M \cdot L}{t^2}$	$\frac{kg \cdot m}{s^2}$	newton (N)
Energy	E	$\frac{M \cdot L^2}{t^2}$	$\frac{kg \cdot m^2}{s^2}$	joule (J), $N \cdot m$
Power	P	$\frac{M \cdot L^2}{t^3}$	$\frac{kg \cdot m^2}{s^3}$	watt (W), $\frac{J}{s}$
Rate of heat transfer	$Q = qA$	$\frac{M \cdot L^2}{t^3}$	$\frac{kg \cdot m^2}{s^3}$	watt (W), $\frac{J}{s}$
Heat flux	q	$\frac{M}{t^3}$	$\frac{kg}{s^3}$	$\frac{W}{m^2}, \frac{J}{s \cdot m^2}$
Heat generation rate per unit volume	$\dot{q}$	$\frac{M}{L \cdot t^3}$	$\frac{kg}{m \cdot s^3}$	$\frac{W}{m^3}, \frac{J}{s \cdot m^3}$
Temperature	T	T	K	$^\circ C = K - 273.15$
Pressure	P	$\frac{M}{L \cdot t^2}$	$\frac{kg}{m \cdot s^2}$	pascal (Pa), $\frac{N}{m^2}$
Velocity	u, v, w	$\frac{L}{t}$	$\frac{m}{s}$	
Density	ρ	$\frac{M}{L^3}$	$\frac{kg}{m^3}$	
Thermal conductivity	k	$\frac{M \cdot L}{t^3 \cdot T}$	$\frac{kg \cdot m}{s^3 \cdot K}$	$\frac{W}{m \cdot K}$
Specific heat	c	$\frac{L^2}{t^2 \cdot T}$	$\frac{m^2}{s^2 \cdot K}$	$\frac{J}{kg \cdot K}$
Dynamic (absolute) viscosity	μ	$\frac{M}{L \cdot t}$	$\frac{kg}{m \cdot s}$	$Pa \cdot s$, $\frac{N \cdot s}{m^2}$
Thermal diffusivity	$\alpha = \frac{k}{\rho c}$	$\frac{L^2}{t}$	$\frac{m^2}{s}$	
Kinematic Viscosity	$\nu = \frac{\mu}{\rho}$	$\frac{L^2}{t}$	$\frac{m^2}{s}$	
Convective heat transfer coefficient	h	$\frac{M}{t^3 \cdot T}$	$\frac{kg}{s^3 \cdot K}$	$\frac{W}{m^2 \cdot K}, \frac{J}{s \cdot m^2 \cdot K}$

Cartesian Tensor Notation (Einstein Convention)

Cartesian tensor notation is a compact method for writing equations. A few simple rules can be used to expand an equation into is full form based on the subscript indices. The range of the indices are based on the spatial dimension of the problem. If an index is repeated within a term of the equation, then a summation over the index is implied.
Two-dimensions:

$$q_i n_i = q_1 n_1 + q_2 n_2 = q_x n_x + q_y n_y$$

Three-dimensions:

$$q_i n_i = q_1 n_1 + q_2 n_2 + q_3 n_3 = q_x n_x + q_y n_y + q_z n_z$$

References I

[BLID11] T.L. Bergman, A.S. Lavine, F.P. Incropera, and D.P. DeWitt.
Introduction to Heat Transfer.
John Wiley & Sons, New York, sixth edition, 2011.

[LL12] J. H. Lienhard, IV and J. H. Lienhard, V.
A Heat Transfer Textbook.
Phlogiston Press, Cambridge, Massachusetts, fourth edition, 2012.
Available at: `http://ahtt.mit.edu`.