Thermal Network Analysis with TNSolver

Steady Conduction - The Composite Wall Problem

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Outline

Heat Transfer Analysis using Thermal Networks

- Heat Transfer Math Model
  - Steady, plane wall conduction with convection
  - Control volumes and the integral form
- TNSolver User Guide
- Composite Wall Problem

Heat Transfer

Math Model

Conduction, Convection and Radiation

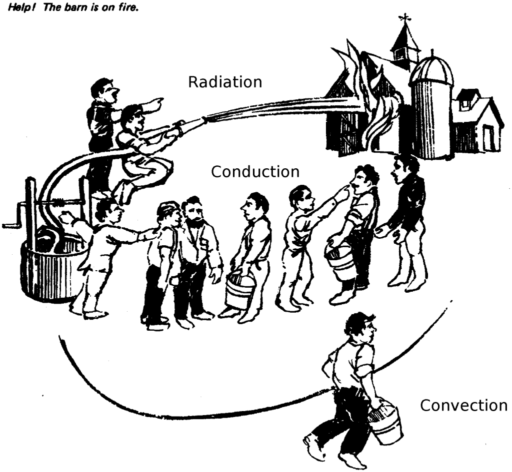


Figure borrowed from [LL12].

Heat Transfer Analysis

Math Model

Answering design questions about thermal energy and temperature

- Hand calculation - back-of-the-envelope
  - On the order of 1-10 equations
- Thermal network or lumped parameter approach
  - On the order of 10-1,000 equations
- Continuum approach - solid model/mesh generation
  - On the order of 1,000-1,000,000 equations
  - Finite Volume Method (FVM)
  - Finite Element Method (FEM)

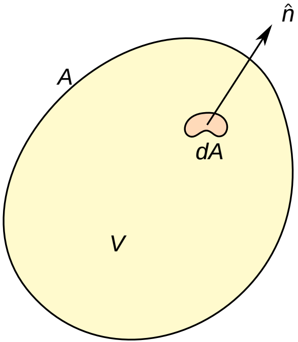
See Section 1.5, page 38, in [BLID11]

The Control Volume Concept

Math Model

$$\sum \text{Energy In} - \sum \text{Energy Out} =$$
$$\text{Energy Stored, Generated and/or Consumed}$$

Heat (transfer) is thermal energy transfer due to a temperature difference



Integral Form of Steady Heat Conduction

Math Model

The steady conduction equation, in Cartesian tensor integral form, is:

$$\int_A q_i n_i \, dA = \iint_V \dot{q} \, dV$$

where $\dot{q}$ is a volumetric source and Fourier's Law of Heat Conduction provides a constitutive model for the heat flux as a function of temperature gradient:

$$q_i = -k \frac{\partial T}{\partial x_i}$$

where k is the isotropic thermal conductivity.

Convection

Math Model

Convection heat transfer from the surface of the control volume is modeled by:

$$\int_{\Gamma_c} q_i n_i \, dA = \int_{\Gamma_c} h(T_s - T_c)\, dA, \quad \text{where} \left\{ \begin{array}{ll} T_s > T_c, & \text{cooling} \\ T_s < T_c, & \text{heating} \end{array} \right.$$

The convection coefficient, $h(x_i, t, T_s, T_c)$, is usually a function of position, time, surface temperature, T_s, free stream or bulk temperature, T_c, and other parameters. The value of the coefficient is often evaluated using a correlation.

Introducing TNSolver

- Thermal Network Solver - TNSolver
- MATLAB/Octave program
  - GNU Octave is an open source clone of MATLAB
- Thermal model is described in a text input file
  - For example: edit using notepad on Windows
- Simulation results are both returned from the function and written to text output files for post-processing

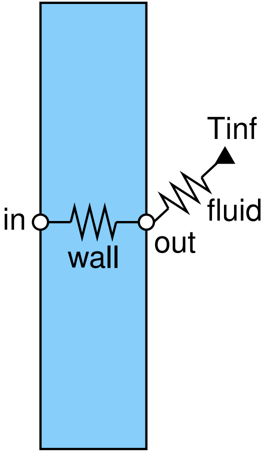
Thermal Network Terminology

TNSolver User Guide

- Time dependency
  - Steady state or transient
  - Initial condition is required for transient
- Geometry
  - Control Volume - volume, $V = \int_V dV$
    - Node: ●, $T_{\text{node}} = \int_V T(x_i)dV$
  - Control Volume Surface - area, $A = \int_A dA$
    - Surface Node: ○, $T_{\text{surface node}} = \int_A T(x_i)dA$
- Material properties
- Conductors
  - Conduction
  - Convection
  - Radiation
- Boundary conditions
  - Boundary node: ▲
- Sources/sinks

TNSolver Input Example of Text Input File


```
!  Simple Wall Model

Begin Solution Parameters
  type  = steady
End Solution Parameters

Begin Conductors
 wall  conduction in  out   2.3  1.2   1.0 ! k L A
 fluid convection out Tinf  2.3  1.0       ! h A
End Conductors

Begin Boundary Conditions
  fixed_T  21.0   in    ! Inner wall T
  fixed_T  5.0    Tinf  ! Fluid T
End Boundary Conditions
```



! begins a comment (MATLAB uses %)

Solution Parameters

TNSolver User Guide

```
Begin Solution Parameters

  type  = steady  ! <steady|transient>
  units = SI      ! <SI|US>

End Solution Parameters
```

Conduction: Cartesian (The Plane Wall)

TNSolver User Guide

The rate of heat transfer, Q_{ij}, due to conduction, between the two temperatures T_i and T_j, separated by a distance L and area A, is:

$$Q_{ij} = \frac{kA}{L}\left(T_i - T_j\right)$$

The heat flux, q_{ij}, is:

$$q_{ij} = \frac{Q_{ij}}{A} = \frac{k}{L}\left(T_i - T_j\right)$$

```
Begin Conductors

! label  type        node i node j  parameters
  name conduction  label   label    x.x x.x x.x  ! k L A

End Conductors
```

Convection Conductor

TNSolver User Guide

The rate of heat transfer due to convection is:

$$Q_{ij} = hA\left(T_i - T_j\right)$$

```
Begin Conductors

! label  type        node i node j  parameters
  name convection  label  label    x.x x.x    ! h A

End Conductors
```

Specified Surface Temperature Boundary Condition

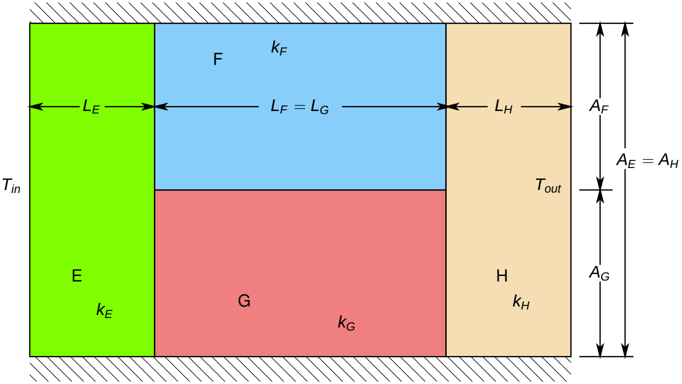
The surface node temperature, T_b is specified:

```
Begin Boundary Conditions

! type     parameter(s)   node(s)
 fixed_T   T_b            label

End Boundary Conditions
```

Description of the Composite Wall Problem

Composite Wall Model

Consider a composite wall:



See Figure 3.3, on page 117, in [BLID11].

Model Parameters

Composite Wall Model

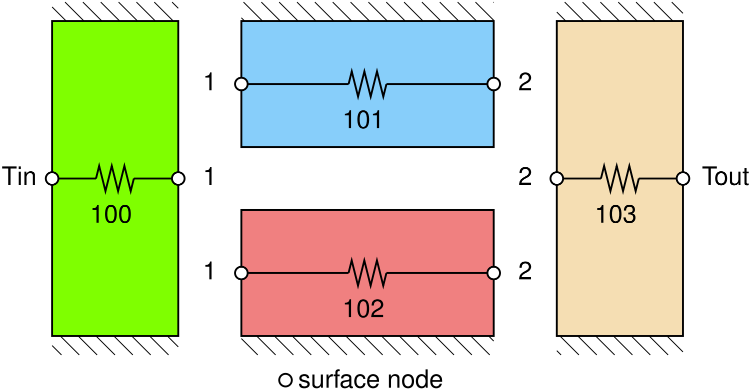
The inner wall temperature $T_{in} = 1$

The outer wall temperature $T_{out} = 0$

Region	Conductivity, k	Length, L	Area, A
E	1.0	1.0	2.0
F	2.0	2.0	1.0
G	$0.001 \leq k_G \leq 2.0$	2.0	1.0
H	3.0	1.0	2.0

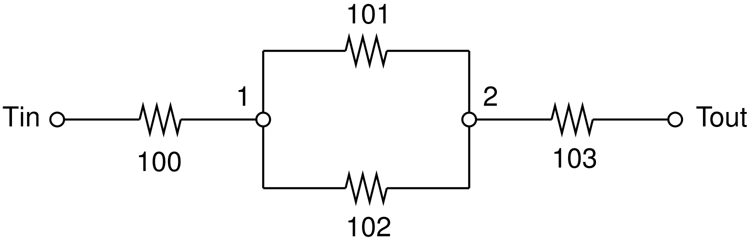
First Approach
Composite Wall Model 1

There are four control volumes:



Network Diagram

Composite Wall Model 1



Compare with Figure 3.3 (a), on page 117, in [BLID11].

TNSolver Input File for $k_G = 2.0$

Composite Wall Model 1

```
!  Composite wall model: Approach 1 series-parallel

Begin Solution Parameters

  type   = steady

End Solution Parameters

Begin Conductors

! label  type        node 1   node 2   parameters
   100   conduction   Tin      1       1.0   1.0   2.0 ! k_E L_E A_E
   101   conduction   1        2       2.0   2.0   1.0 ! k_F L_F A_F
   102   conduction   1        2       2.0   2.0   1.0 ! k_G L_G A_G
   103   conduction   2       Tout     3.0   1.0   2.0 ! k_H L_H A_H

End Conductors

Begin Boundary Conditions

! type    parameter(s)   node(s)
 fixed_T     1.0          Tin     ! inner wall temperature
 fixed_T     0.0          Tout    ! outer wall temperature

End Boundary Conditions
```

TNSolver Output for $k_G = 2.0$

Composite Wall Model 1

Nodes

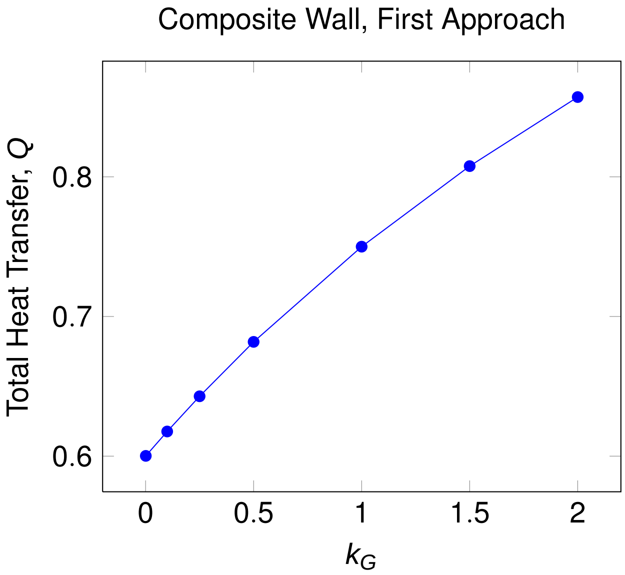
Label	Material	Volume (m^3)	Temperature (C)
Tin	N/A	0	1
1	N/A	0	0.571429
2	N/A	0	0.142857
Tout	N/A	0	0

Conductors

Label	Type	Node i	Node j	Q_ij (W)
100	conduction	Tin	1	0.857143
101	conduction	1	2	0.428571
102	conduction	1	2	0.428571
103	conduction	2	Tout	0.857143

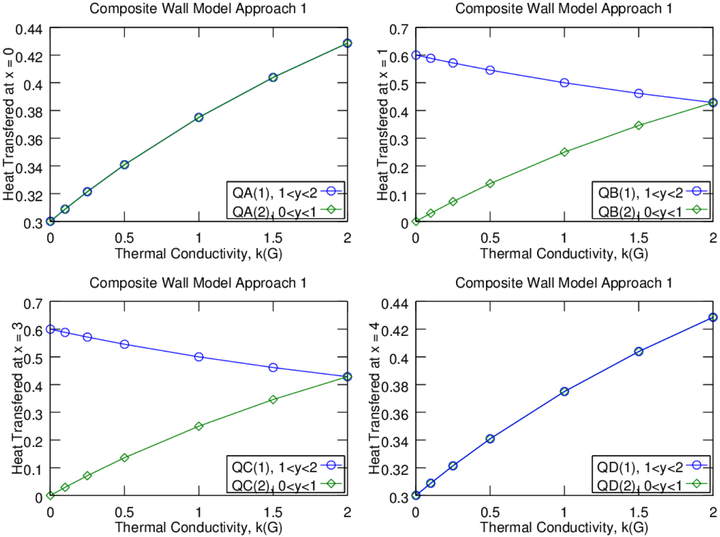
Total Heat Transfer over the Range of k_G

Composite Wall Model 1



Total Heat Transfer at Region Interfaces

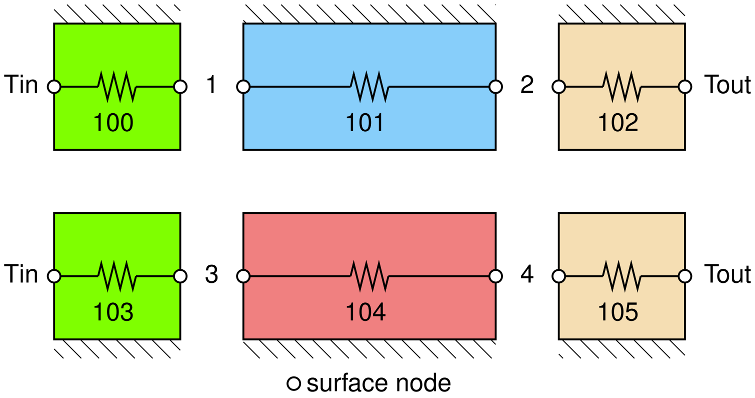
Composite Wall Model 1



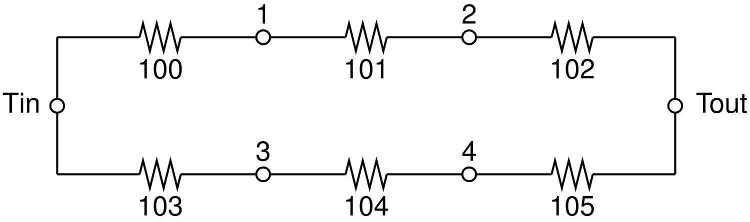
Second Approach

Composite Wall Model 2

There are six control volumes:



Network Diagram
Composite Wall Model 2



Compare with Figure 3.3 (b), on page 117, in [BLID11].

TNSolver Input File for $k_G = 2.0$

Composite Wall Model 2

```
!  Composite wall model: Approach 2 - parallel conductors

Begin Solution Parameters

  type   = steady

End Solution Parameters

Begin Conductors

! label  type        node 1  node 2  parameters
   100   conduction   Tin     1        1.0, 1.0, 1.0  ! k_E L_E A_E
   101   conduction   1       2        2.0, 2.0, 1.0  ! k_F L_F A_F
   102   conduction   2       Tout     3.0, 1.0, 1.0  ! k_H L_H A_H
   103   conduction   Tin     3        1.0, 1.0, 1.0  ! k_E L_E A_E
   104   conduction   3       4        2.0, 2.0, 1.0  ! k_G L_G A_G
   105   conduction   4       Tin      3.0, 1.0, 1.0  ! k_H L_H A_H

End Conductors

Begin Boundary Conditions

! type    parameter(s)   node(s)
 fixed_T      1.0          Tin     ! inner wall temperature
 fixed_T      0.0          Tout    ! outer wall temperature

End Boundary Conditions
```

TNSolver Output for $k_G = 2.0$

Composite Wall Model 2

Nodes

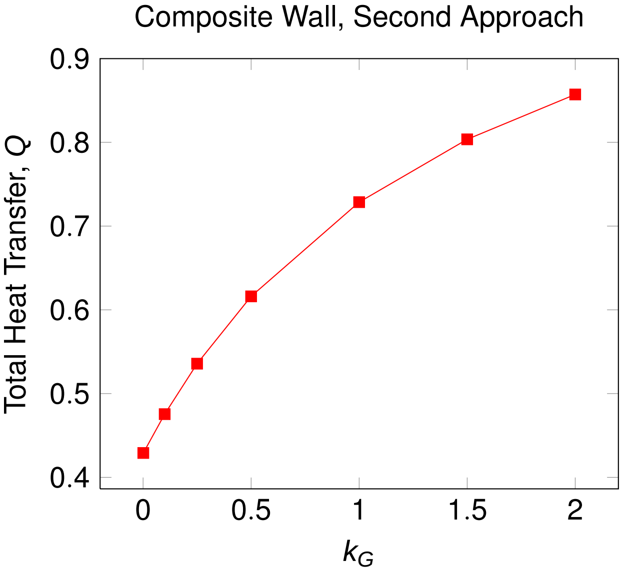
Label	Material	Volume (m^3)	Temperature (C)
T1	N/A	0	1
1	N/A	0	0.571429
2	N/A	0	0.142857
T2	N/A	0	0
3	N/A	0	0.571429
4	N/A	0	0.142857

Conductors

Label	Type	Node i	Node j	Q_ij (W)
100	conduction	T1	1	0.428571
101	conduction	1	2	0.428571
102	conduction	2	T2	0.428571
103	conduction	T1	3	0.428571
104	conduction	3	4	0.428571
105	conduction	4	T2	0.428571

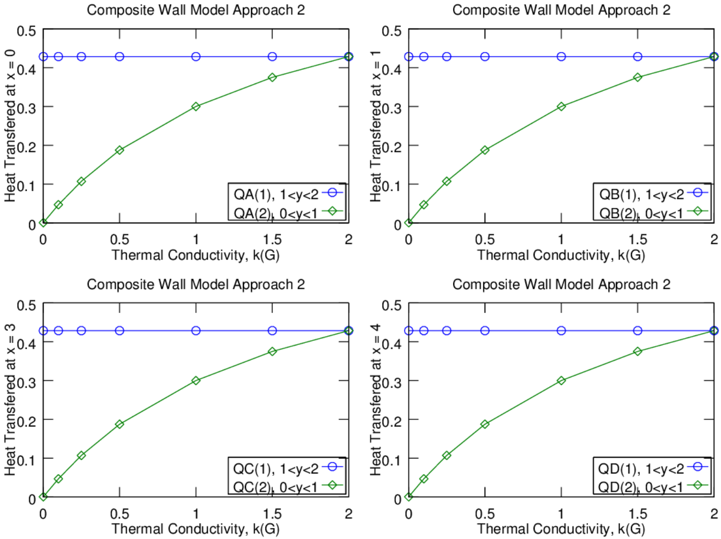
Total Heat Transfer over the Range of k_G

Composite Wall Model 2



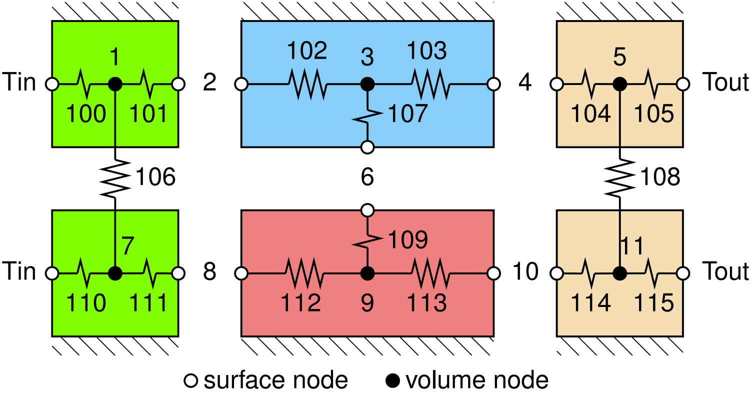
Total Heat Transfer at Region Interfaces

Composite Wall Model 2



Third Approach
Composite Wall Model 3

There are six control volumes:



TNSolver Input File for $k_G = 2.0$

Composite Wall Model 3

```
!  Composite wall model: Approach 3

Begin Solution Parameters

  type  = steady

End Solution Parameters

Begin Conductors

! label  type       node 1  node 2  parameters
   100  conduction   Tin     1        1.0, 0.5, 1.0  ! k_E L_E A_E/2
   101  conduction   1       2        1.0, 0.5, 1.0  ! k_E L_E A_E/2
   102  conduction   2       3        2.0, 1.0, 1.0  ! k_F L_F A_F
   103  conduction   3       4        2.0, 1.0, 1.0  ! k_F L_F A_F
   104  conduction   4       5        3.0, 0.5, 1.0  ! k_H L_H A_H/2
   105  conduction   5      Tout      3.0, 0.5, 1.0  ! k_H L_H A_H/2

   106  conduction   1       7        1.0, 1.0, 1.0  ! k_E
   107  conduction   3       6        2.0, 0.5, 2.0  ! k_F
   108  conduction   5      11        3.0, 1.0, 1.0  ! k_H
   109  conduction   6       9        2.0, 0.5, 2.0  ! k_G

   110  conduction   Tin     7        1.0, 0.5, 1.0  ! k_E L_E A_E/2
   111  conduction   7       8        1.0, 0.5, 1.0  ! k_E L_E A_E/2
   112  conduction   8       9        2.0, 1.0, 1.0  ! k_G L_G A_G
   113  conduction   9      10        2.0, 1.0, 1.0  ! k_G L_G A_G
   114  conduction  10      11        3.0, 0.5, 1.0  ! k_H L_H A_H/2
   115  conduction  11      Tout      3.0, 0.5, 1.0  ! k_H L_H A_H/2

End Conductors

Begin Boundary Conditions

! type    parameter(s)   node(s)
 fixed_T      1.0          Tin    ! inner wall temperature
 fixed_T      0.0          Tout   ! outer wall temperature

End Boundary Conditions
```

TNSolver Output for $k_G = 2.0$

Composite Wall Model 3

Nodes

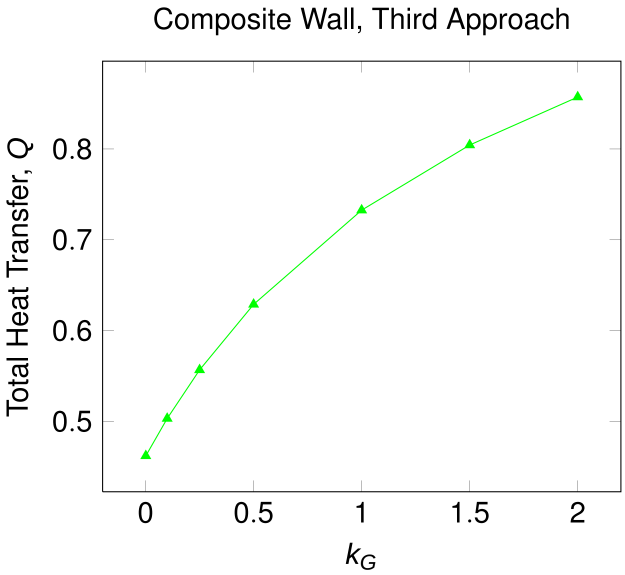
Label	Material	Volume (m^3)	Temperature (C)
Tin	N/A	0	1
1	N/A	0	0.785714
2	N/A	0	0.571429
3	N/A	0	0.357143
4	N/A	0	0.142857
5	N/A	0	0.0714286
Tout	N/A	0	-1.13687e-013
7	N/A	0	0.785714
6	N/A	0	0.357143
11	N/A	0	0.0714286
9	N/A	0	0.357143
8	N/A	0	0.571429
10	N/A	0	0.142857

Conductors

Label	Type	Node i	Node j	Q_ij (W)
100	conduction	Tin	1	0.428571
101	conduction	1	2	0.428571
102	conduction	2	3	0.428571
103	conduction	3	4	0.428571
104	conduction	4	5	0.428571
105	conduction	5	Tout	0.428571
106	conduction	1	7	-5.68434e-014
107	conduction	3	6	0
108	conduction	5	11	-3.41061e-013
109	conduction	6	9	-4.54747e-013
110	conduction	Tin	7	0.428571
111	conduction	7	8	0.428571
112	conduction	8	9	0.428571
113	conduction	9	10	0.428571
114	conduction	10	11	0.428571
115	conduction	11	Tout	0.428571

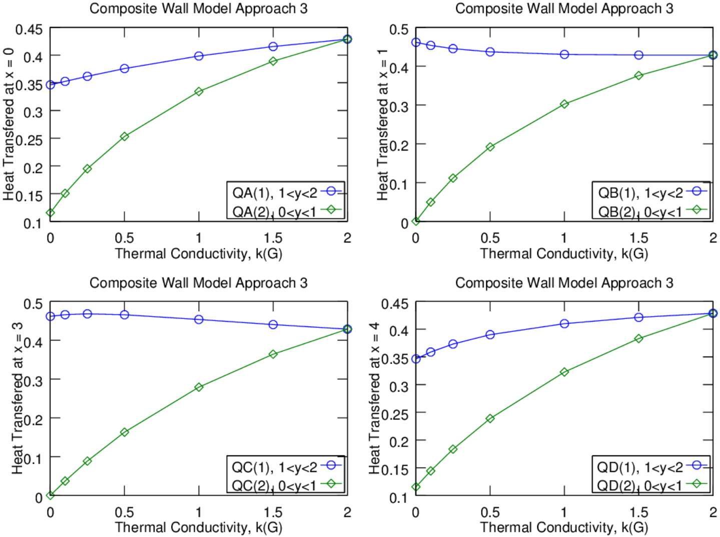
Total Heat Transfer over the Range of k_G

Composite Wall Model 3



Total Heat Transfer at Region Interfaces

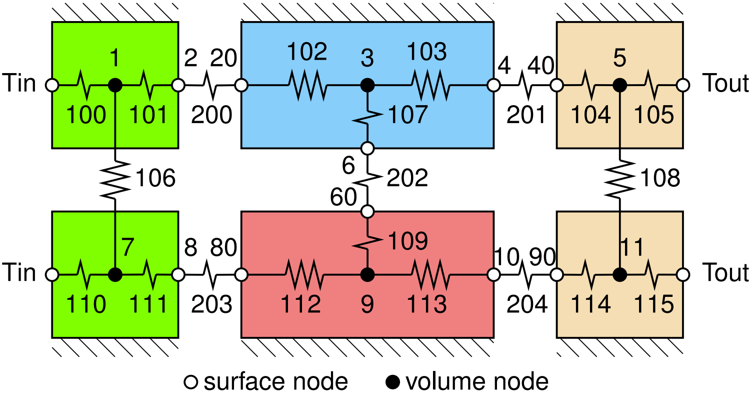
Composite Wall Model 3



Including Contact Resistance

Composite Wall Model 3 with Contact Resistance

There are six control volumes:



Contact resistance between the regions is $h = 0.10$

TNSolver Input File for $k_G = 2.0$

Composite Wall Model 3 with Contact Resistance

```
!  Composite wall model: Approach 3 with Contact Resistance

Begin Solution Parameters

  type  = steady
  units = SI

End Solution Parameters

Begin Conductors

! label  type       node 1  node 2  parameters
   100  conduction   Tin     1       1.0, 0.5, 1.0  ! k_E L_E A_E/2
   101  conduction   1       2       1.0, 0.5, 1.0  ! k_E L_E A_E/2
   102  conduction   20      3       2.0, 1.0, 1.0  ! k_F L_F A_F
   103  conduction   3       4       2.0, 1.0, 1.0  ! k_F L_F A_F
   104  conduction   40      5       3.0, 0.5, 1.0  ! k_H L_H A_H/2
   105  conduction   5      Tout     3.0, 0.5, 1.0  ! k_H L_H A_H/2

   106  conduction   1       7       1.0, 1.0, 1.0  ! k_E
   107  conduction   3       6       2.0, 0.5, 2.0  ! k_F
   108  conduction   5      11       3.0, 1.0, 1.0  ! k_H
   109  conduction   60      9       2.0, 0.5, 2.0  ! k_G

   110  conduction   Tin     7       1.0, 0.5, 1.0  ! k_E L_E A_E/2
   111  conduction   7       8       1.0, 0.5, 1.0  ! k_E L_E A_E/2
   112  conduction   80      9       2.0, 1.0, 1.0  ! k_G L_G A_G
   113  conduction   9      10       2.0, 1.0, 1.0  ! k_G L_G A_G
   114  conduction   90     11       3.0, 0.5, 1.0  ! k_H L_H A_H/2
   115  conduction   11     Tout     3.0, 0.5, 1.0  ! k_H L_H A_H/2

   200  convection   2      20       0.1  1.0   ! h contact resistance
   201  convection   4      40       0.1  1.0   ! h contact resistance
   202  convection   6      60       0.1  2.0   ! h contact resistance
   203  convection   8      80       0.1  1.0   ! h contact resistance
   204  convection   10     90       0.1  1.0   ! h contact resistance

End Conductors

Begin Boundary Conditions

! type    parameter(s)   node(s)
 fixed_T      1.0          Tin    ! inner wall temperature
 fixed_T      0.0          Tout   ! outer wall temperature

End Boundary Conditions
```

TNSolver Output for $k_G = 2.0$

Composite Wall Model 3 with Contact Resistance

Nodes

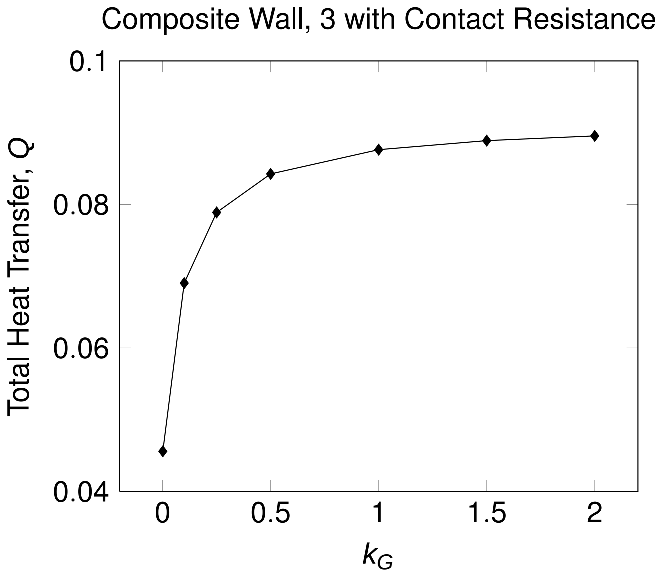
Label	Material	Volume (m^3)	Temperature (C)
Tin	N/A	0	1
1	N/A	0	0.977612
2	N/A	0	0.955224
20	N/A	0	0.507463
3	N/A	0	0.485075
4	N/A	0	0.462687
40	N/A	0	0.0149254
5	N/A	0	0.00746269
Tout	N/A	0	1.13687e-013
7	N/A	0	0.977612
6	N/A	0	0.485075
11	N/A	0	0.00746269
60	N/A	0	0.485075
9	N/A	0	0.485075
8	N/A	0	0.955224
80	N/A	0	0.507463
10	N/A	0	0.462687
90	N/A	0	0.0149254

Conductors

Label	Type	Node i	Node j	Q_ij (W)
100	conduction	Tin	1	0.0447761
101	conduction	1	2	0.0447761
102	conduction	20	3	0.0447761
103	conduction	3	4	0.0447761
104	conduction	40	5	0.0447761
105	conduction	5	Tout	0.0447761
106	conduction	1	7	0
107	conduction	3	6	-4.54747e-013
108	conduction	5	11	0
109	conduction	60	9	0
110	conduction	Tin	7	0.0447761
111	conduction	7	8	0.0447761
112	conduction	80	9	0.0447761
113	conduction	9	10	0.0447761
114	conduction	90	11	0.0447761
115	conduction	11	Tout	0.0447761
200	convection	2	20	0.0447761
201	convection	4	40	0.0447761
202	convection	6	60	5.68434e-014
203	convection	8	80	0.0447761
204	convection	10	90	0.0447761

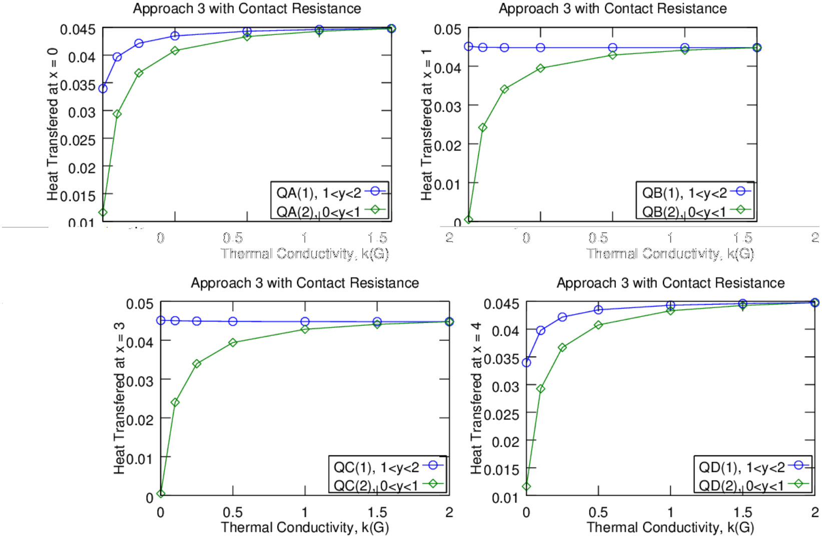
Total Heat Transfer over the Range of k_G

Composite Wall Model 3 with Contact Resistance



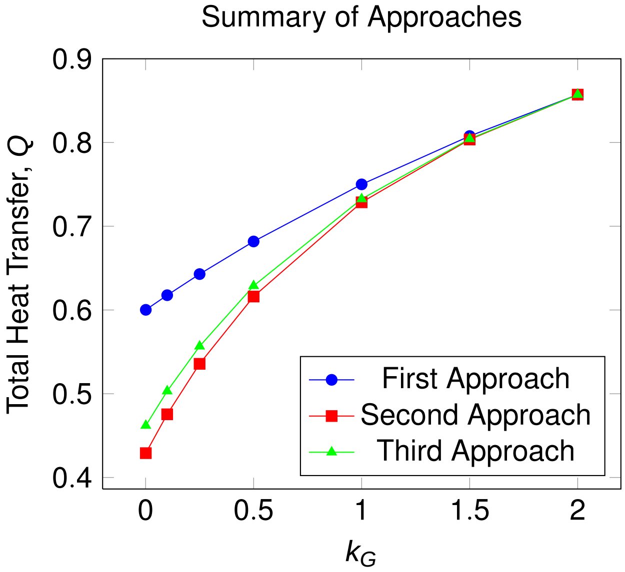
Total Heat Transfer at Region Interfaces

Composite Wall Model 3 with Contact Resistance



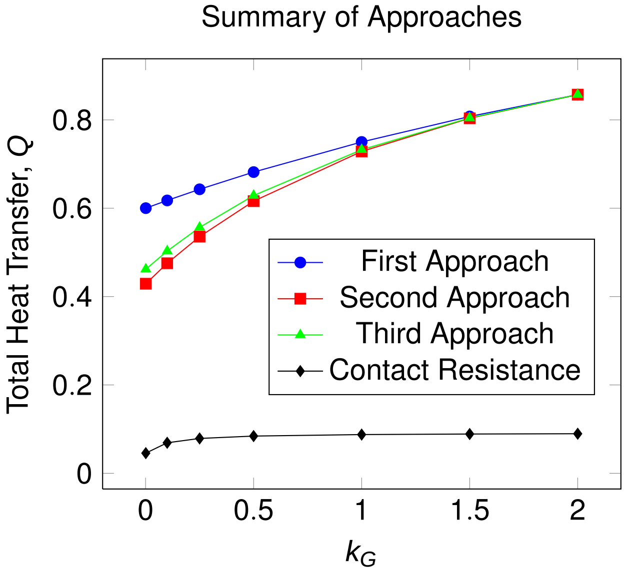
Total Heat Transfer over the Range of k_G

Composite Wall Model Summary



Total Heat Transfer over the Range of k_G

Composite Wall Model Summary



Conclusion

Questions?

Obtaining GNU Octave

GNU Octave

- GNU Octave
  - http://www.gnu.org/software/octave/
- Octave Wiki
  - http://wiki.octave.org
- Octave-Forge Packages (similar to MATLAB Toolbox packages)
  - http://octave.sourceforge.net
- For Windows installation I would suggest the MinGW installation.
  - If you already have Cygwin installed, then install that version.

SI Units

Quantity	Symbol		Fundamental	Derivatives
Mass	m	M	kg	
Length	x, y, z	L	m	
Area	A	L^2	m^2	
Volume	V	L^3	m^3	
Time	t	t	s	
Force	F	$\frac{M \cdot L}{t^2}$	$\frac{kg \cdot m}{s^2}$	newton (N)
Energy	E	$\frac{M \cdot L^2}{t^2}$	$\frac{kg \cdot m^2}{s^2}$	joule (J), $N \cdot m$
Power	P	$\frac{M \cdot L^2}{t^3}$	$\frac{kg \cdot m^2}{s^3}$	watt (W), $\frac{J}{s}$
Rate of heat transfer	$Q = qA$	$\frac{M \cdot L^2}{t^3}$	$\frac{kg \cdot m^2}{s^3}$	watt (W), $\frac{J}{s}$
Heat flux	q	$\frac{M}{t^3}$	$\frac{kg}{s^3}$	$\frac{W}{m^2}, \frac{J}{s \cdot m^2}$
Heat generation rate per unit volume	$\dot{q}$	$\frac{M}{L \cdot t^3}$	$\frac{kg}{m \cdot s^3}$	$\frac{W}{m^3}, \frac{J}{s \cdot m^3}$
Temperature	T	T	K	$^\circ C = K - 273.15$
Pressure	P	$\frac{M}{L \cdot t^2}$	$\frac{kg}{m \cdot s^2}$	pascal (Pa), $\frac{N}{m^2}$
Velocity	u, v, w	$\frac{L}{t}$	$\frac{m}{s}$	
Density	ρ	$\frac{M}{L^3}$	$\frac{kg}{m^3}$	
Thermal conductivity	k	$\frac{M \cdot L}{t^3 \cdot T}$	$\frac{kg \cdot m}{s^3 \cdot K}$	$\frac{W}{m \cdot K}$
Specific heat	c	$\frac{L^2}{t^2 \cdot T}$	$\frac{m^2}{s^2 \cdot K}$	$\frac{J}{kg \cdot K}$
Dynamic (absolute) viscosity	μ	$\frac{M}{L \cdot t}$	$\frac{kg}{m \cdot s}$	$Pa \cdot s$, $\frac{N \cdot s}{m^2}$
Thermal diffusivity	$\alpha = \frac{k}{\rho c}$	$\frac{L^2}{t}$	$\frac{m^2}{s}$	
Kinematic Viscosity	$\nu = \frac{\mu}{\rho}$	$\frac{L^2}{t}$	$\frac{m^2}{s}$	
Convective heat transfer coefficient	h	$\frac{M}{t^3 \cdot T}$	$\frac{kg}{s^3 \cdot K}$	$\frac{W}{m^2 \cdot K}, \frac{J}{s \cdot m^2 \cdot K}$

US Customary Units

Quantity	Symbol		Fundamental	Derivatives
Mass	m	M	lb_m	
Length	x, y, z	L	ft	
Area	A	L^2	ft^2	
Volume	V	L^3	ft^3	
Time	t	t	s	
Force	F	$\frac{M\cdot L}{t^2}$	$\frac{lb_m\cdot ft}{s^2}$	lb_f
Energy	E	$\frac{M\cdot L^2}{t^2}$	$\frac{lb_m\cdot ft^2}{s^2}$	Btu
Power	P	$\frac{M\cdot L^2}{t^3}$	$\frac{lb_m\cdot ft^2}{s^3}$	hp
Rate of heat transfer	$Q = qA$	$\frac{M\cdot L^2}{t^3}$	$\frac{lb_m\cdot ft^2}{s^3}$	$\frac{Btu}{s}$
Heat flux	q	$\frac{M}{t^3}$	$\frac{lb_m}{s^3}$	$\frac{Btu}{s\cdot ft^2}$
Heat generation rate per unit volume	$\dot{q}$	$\frac{M}{L\cdot t^3}$	$\frac{lb_m}{ft\cdot s^3}$	$\frac{Btu}{s\cdot ft^3}$
Temperature	T	T	$^\circ R$	$^\circ F = {}^\circ R - 459.67$
Pressure	P	$\frac{M}{L\cdot t^2}$	$\frac{lb_m}{ft\cdot s^2}$	
Velocity	u, v, w	$\frac{L}{t}$	$\frac{ft}{s}$	
Density	ρ	$\frac{M}{L^3}$	$\frac{lb_m}{ft^3}$	
Thermal conductivity	k	$\frac{M\cdot L}{t^3\cdot T}$	$\frac{lb_m\cdot ft}{s^3\cdot\ R}$	$\frac{Btu}{s\cdot ft\cdot\ R}$
Specific heat	c	$\frac{L^2}{t^2\cdot T}$	$\frac{ft^2}{s^2\cdot\ R}$	$\frac{Btu}{lb_m\cdot\ R}$
Dynamic (absolute) viscosity	μ	$\frac{M}{L\cdot t}$	$\frac{lb_m}{ft\cdot s}$	
Thermal diffusivity	$\alpha = \frac{k}{\rho c}$	$\frac{L^2}{t}$	$\frac{ft^2}{s}$	
Kinematic Viscosity	$\nu = \frac{\mu}{\rho}$	$\frac{L^2}{t}$	$\frac{ft^2}{s}$	
Convective heat transfer coefficient	h	$\frac{M}{t^3\cdot T}$	$\frac{lb_m}{s^3\cdot\ R}$	$\frac{Btu}{s\cdot ft^2\cdot\ R}$

Cartesian Tensor Notation (Einstein Convention)

Cartesian tensor notation is a compact method for writing equations. A few simple rules can be used to expand an equation into is full form based on the subscript indices. The range of the indices are based on the spatial dimension of the problem. If an index is repeated within a term of the equation, then a summation over the index is implied.

Two-dimensions:

$$q_i n_i = q_1 n_1 + q_2 n_2 = q_x n_x + q_y n_y$$

Three-dimensions:

$$q_i n_i = q_1 n_1 + q_2 n_2 + q_3 n_3 = q_x n_x + q_y n_y + q_z n_z$$

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