The Cannon Problem

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ME498/599 Ballistic Problem with Air Resistance

- Sensitivity analysis
- Uncertainty quantification
- Uncertainty propagation
The equations of motion (Newton’s Second Law: \( ma = \sum F \)) are:

- \( \frac{d^2x(t)}{dt^2} = -\frac{D_f}{m} \frac{dx(t)}{dt} = -\frac{D_f}{m} Vv_x \)
- \( \frac{d^2y(t)}{dt^2} = -\frac{D_f}{m} \frac{dy(t)}{dt} - g = -\frac{D_f}{m} Vv_y - g \)
- x-direction velocity: \( v_x = \frac{dx(t)}{dt} \text{ (m/s)} \)
- y-direction velocity: \( v_y = \frac{dy(t)}{dt} \text{ (m/s)} \)
- x-direction acceleration: \( a_x = \frac{d^2x(t)}{dt^2} \text{ (m/s}^2) \)
- y-direction acceleration: \( a_y = \frac{d^2y(t)}{dt^2} \text{ (m/s}^2) \)
- \( V = \sqrt{v_x^2 + v_y^2} \) is the velocity of the sphere (m/s)
- mass of the sphere: \( m = \rho_s V_s \text{ (kg)} \)
The drag force parameter is: 

\[ D_f = \frac{\rho C_d A}{2} \text{ (kg/m)} \]

- \( \rho \) is the fluid density (kg/m\(^3\))
- \( C_d \) is the drag coefficient (dimensionless)
  - a function of Reynolds number: \( Re = \frac{\rho V D}{\mu} \)
  - \( D \) (m), is the diameter of the sphere
  - \( \mu \) (kg \cdot m/s), is the fluid viscosity
- \( A \) is the cross sectional area (frontal area)
  - For a sphere \( A = \frac{\pi D^2}{4} \) (m\(^2\))
Drag Coefficient Correlation for a Sphere

Drag coefficient correlation for a sphere:

\[
C_d = \frac{24}{Re} + \frac{2.6 \left( \frac{Re}{5.0} \right)^{1.52}}{1 + \left( \frac{Re}{5.0} \right)^{1.52}} + \frac{0.411 \left( \frac{Re}{263,000} \right)^{-8.0}}{1 + \left( \frac{Re}{263,000} \right)^{-8.0}} + \frac{0.25 \left( \frac{Re}{10^6} \right)}{1 + \left( \frac{Re}{10^6} \right)}
\]

valid for \(0.01 \leq Re \leq 10^6\)

Equation (8.83) on p. 624 in [Mor13]\(^1\)

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Correlation Comparison with Data

\[ C_D = \frac{24}{Re} + \frac{2.6}{1 + \left(\frac{Re}{5.0}\right)^{1.52}} + \frac{0.411}{1 + \left(\frac{Re}{263,000}\right)^{0.00}} + \frac{0.25}{1 + \left(\frac{Re}{10^6}\right)^{0.00}} \]
Summary of Math Model Parameters

- **Material Properties**
  - Fluid density $\rho (kg/m^3)$
  - Fluid viscosity $\mu (kg \cdot m/s = Pa \cdot s = N \cdot s/m^2)$
  - Sphere density $\rho_s (kg/m^3)$

- **Geometry**
  - Sphere diameter $D (m)$, volume is $V_s = \frac{4\pi r^3}{3} = \frac{\pi D^3}{6} (m^3)$

- **Initial/Boundary Conditions**
  - Muzzle velocity $V (m/s)$
  - Angle $\theta (^\circ)$, $v_x = \cos(\theta)V$ and $v_y = \sin(\theta)V$

- **Correlation**
  - Drag coefficient $C_d(Re)$, a function of Reynolds number

- **Constants**
  - Gravity $g (m^2/s)$