

# Composite Wall Problem Analysis

## Continuum and Thermal Network Methods

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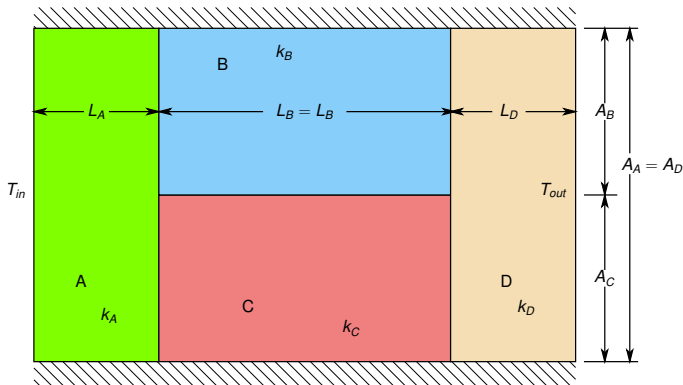
# Outline

- ▶ Composite Wall Problem
- ▶ Continuum Analysis Method
- ▶ Thermal Network Analysis Method

# Description of the Composite Wall Problem

## Composite Wall Model

Consider a composite wall:



See Figure 3.3, on page 117, in [BLID11].

# Model Parameters

## Composite Wall Model

The inner wall temperature  $T_{in} = 100C$

The outer wall temperature  $T_{out} = 0C$

Region	Conductivity, $k$ ( $W/m \cdot K$ )	Length, $L$ ( $m$ )	Area, $A$ ( $m^2$ )
A	1.0	1.0	2.0
B	$0 \leq k_B \leq \infty$	3.0	1.0
C	2.0	3.0	1.0
D	1.0	1.0	2.0

How does the heat flow rate through the wall vary as the thermal conductivity of region  $B$  changes? Compute the heat flow  $q_x$  (or  $Q$ ) and plot as a function of  $k_B$ .

# Solution Verification

## Composite Wall Problem

An analytical solution is used to compare to analysis method results. Only applicable when  $k_B = k_C$ , a one-dimensional problem.

Using Equation (3.19), page 116, in [BLID11]:

$$R_{tot} = \frac{1}{UA} = \frac{L_A}{k_A A_A} + \left[ \frac{k_B A_B}{L_B} + \frac{k_C A_C}{L_C} \right]^{-1} + \frac{L_D}{k_D A_D}$$

$$R_{tot} = \frac{1}{UA} = \frac{1}{(1)(2)} + \left[ \frac{(2)(1)}{3} + \frac{(2)(1)}{3} \right]^{-1} + \frac{1}{(1)(2)} = \frac{7}{4} = 1.75 K/W$$

$$Q = q_x = UA\Delta T = \frac{(T_{in} - T_{out})}{R_{tot}} = \frac{(100.0 - 0.0)}{1.75} = 57.1429 W$$

# Utilize the CHTUNS Solver

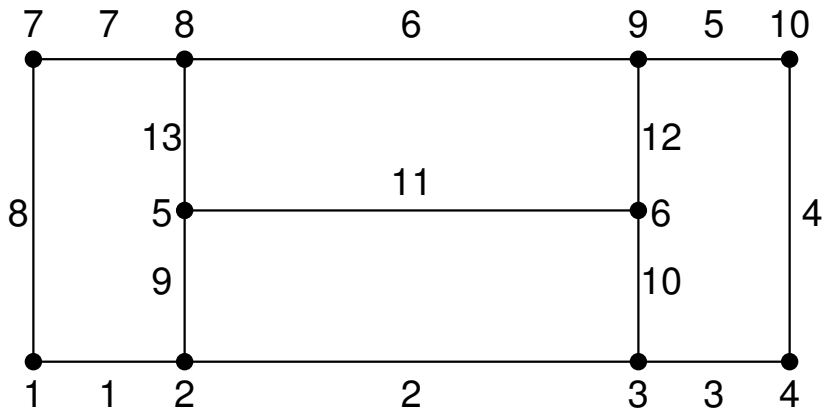
## Continuum Model Analysis

- ▶ CHTUNS: **C**omputational **H**eat **T**ransfer on **UN**structured Meshes
  - ▶ An open source alternative to MATLAB's Partial Differential Equation Toolbox
  - ▶ MATLAB programming language, runs in Octave
  - ▶ Post processing visualization tools included
  - ▶ Solutions presented here utilize a vertex-centered Finite Volume Method
- ▶ Mesh generation utilizing Jonathan Shewchuk's `triangle` program:
  - ▶ <https://www.cs.cmu.edu/~quake/triangle.html>
  - ▶ Use the function `proctriangle.m` to convert `triangle` output to CHTUNS mesh data structure

# Geometry for Mesh Generation

## Continuum Model Analysis

Point and line numbering for planar straight line graph (PSLG)  
input to `triangle` (10 points, 13 lines).



# cwall.poly Input File for triangle

## Continuum Model Analysis

```
# num. points, dimension, num. of attributes, num. of boundary markers
10 2 0 0
# point ID, x, y, <attribute>, <boundary marker>
1 0.0 0.0
2 1.0 0.0
3 4.0 0.0
4 5.0 0.0
5 1.0 1.0
6 4.0 1.0
7 0.0 2.0
8 1.0 2.0
9 4.0 2.0
10 5.0 2.0
# num. of segments, num. of boundary markers
13 4
# segment ID, endpoint, endpoint, <boundary marker>
1 1 2 1
2 2 3 1
3 3 4 1
4 4 10 2
5 10 9 3
6 9 8 3
7 8 7 3
8 7 1 4
9 2 5
10 3 6
11 5 6
12 6 9
13 5 8
# Number of holes
0
# of regional attributes and/or area constraints
4
# <region #> <x> <y> <attribute> <maximum area>
1 0.5 1.0 101
2 2.5 1.5 102
3 2.5 0.5 103
4 4.5 1.0 104
```



# Command to Run `triangle`

## Continuum Model Analysis

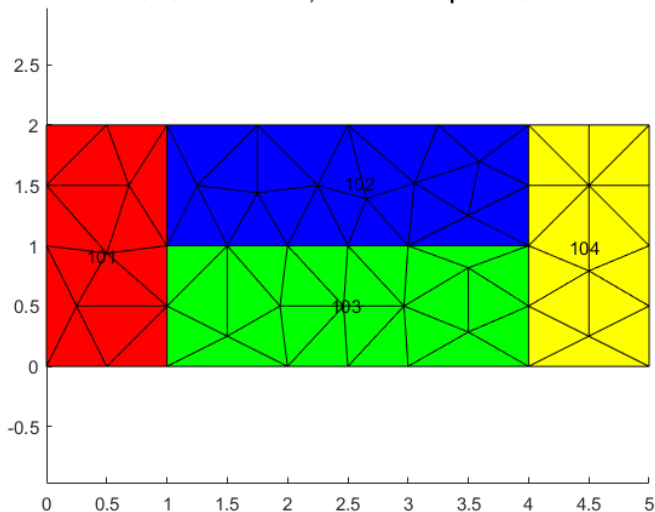
```
triangle -pAqnea0.2 cwall.poly
```

- ▶ `-p` Triangulates a Planar Straight Line Graph (.poly file).
- ▶ `-A` Applies attributes to identify triangles in certain regions.
- ▶ `-q` Quality mesh generation.
- ▶ `-n` Generates a list of triangle neighbors.
- ▶ `-e` Generates an edge list.
- ▶ `-a` Applies a maximum triangle area constraint.

# 50 Node Mesh

## Continuum Model Analysis

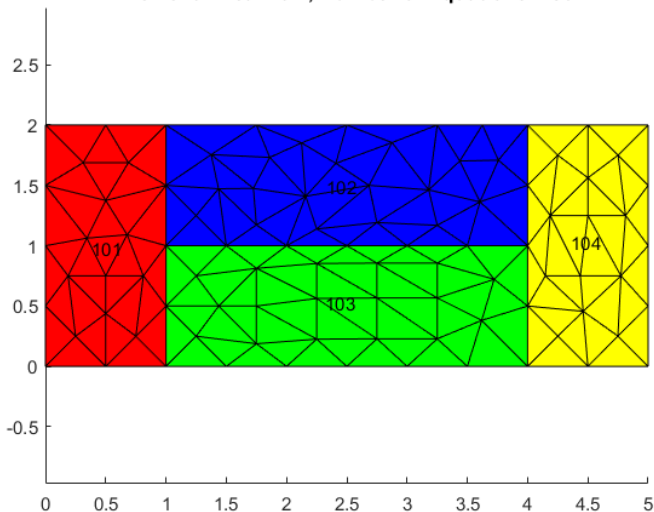
**Element Area = 0.2, Number of Equations = 50**



# 96 Node Mesh

## Continuum Model Analysis

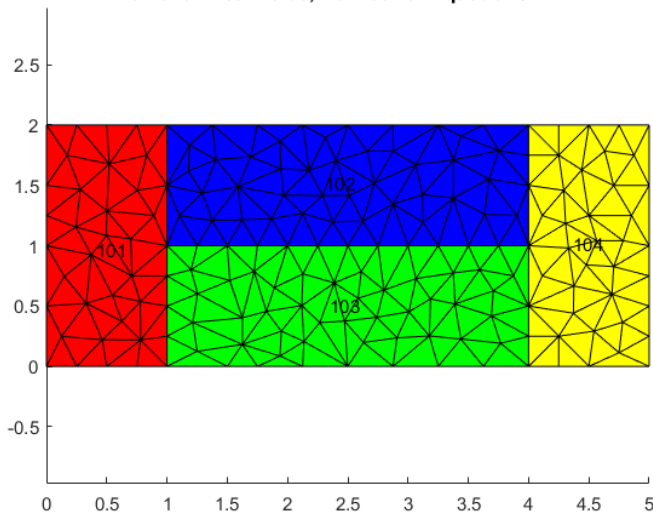
**Element Area = 0.1, Number of Equations = 96**



# 177 Node Mesh

## Continuum Model Analysis

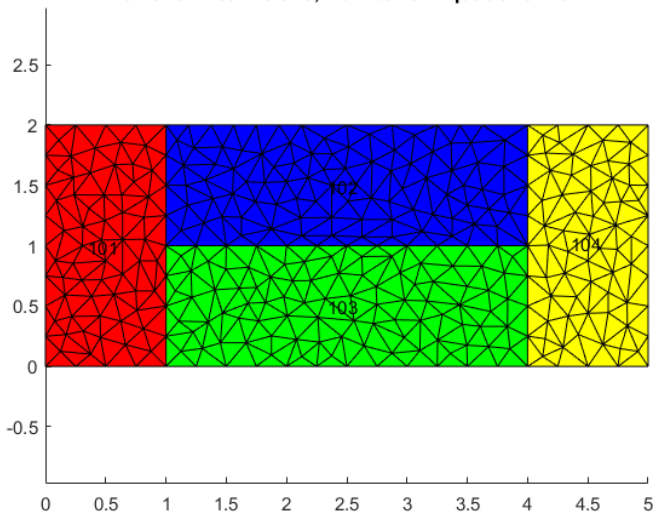
**Element Area = 0.05, Number of Equations = 177**



# 344 Node Mesh

## Continuum Model Analysis

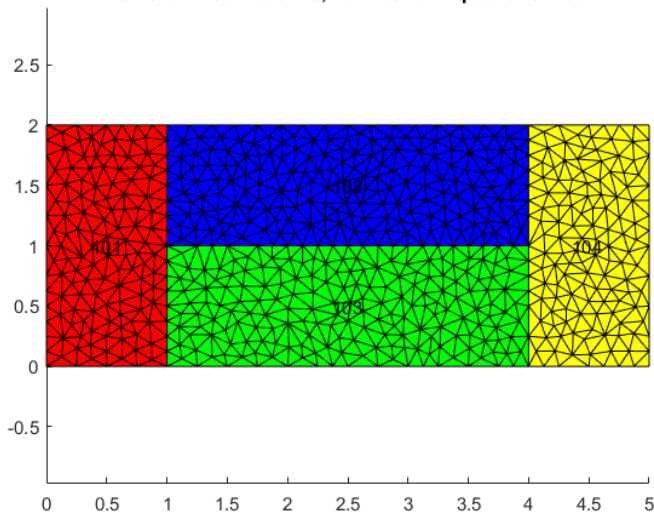
**Element Area = 0.025, Number of Equations = 344**



# 662 Node Mesh

## Continuum Model Analysis

**Element Area = 0.0125, Number of Equations = 662**



# Mesh Convergence Study: Table

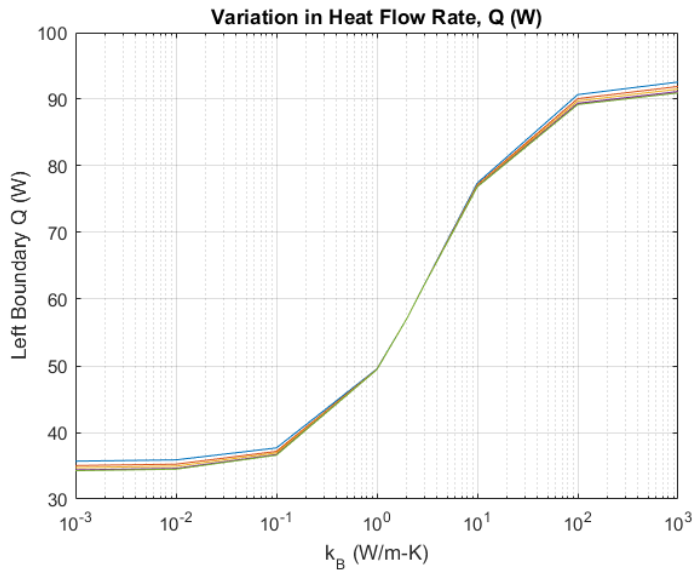
## Continuum Model Analysis

Heat Flow Rate (W)

$k_B$	Mesh Area Parameter				
	0.2	0.1	0.05	0.025	0.0125
0.001	35.6899	35.0308	34.7171	34.3887	34.2625
0.01	35.8802	35.2366	34.9313	34.6118	34.4890
0.1	37.6730	37.1626	36.9273	36.6814	36.5872
1.0	49.5240	49.4669	49.4428	49.4176	49.4075
2.0	57.1429	57.1429	57.1429	57.1429	57.1429
3.0	62.3090	62.2884	62.2791	62.2697	62.2655
10.0	77.4238	77.1660	77.0342	76.9045	76.8375
100.0	90.6939	90.0867	89.7366	89.4018	89.2141
1000.0	92.5833	91.9190	91.5297	91.1587	90.9488

# Mesh Convergence Study: Plot

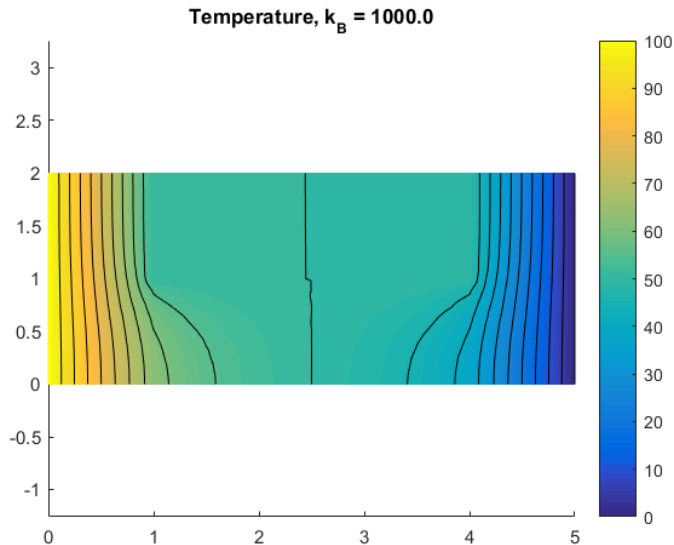
## Continuum Model Analysis





# Temperature Contour Plot

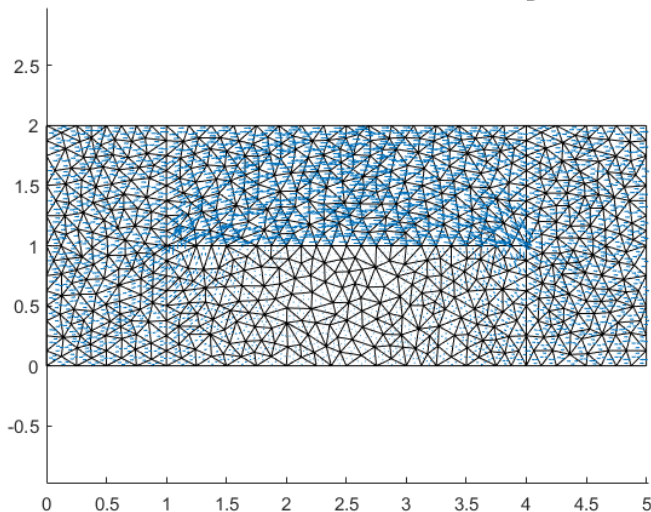
## Continuum Model Analysis



# Heat Flux Vector Plot

## Continuum Model Analysis

Heat Flux Vector at the CV Surface Midpoint,  $k_B = 1000.0$



# Thermal Network Model Analysis

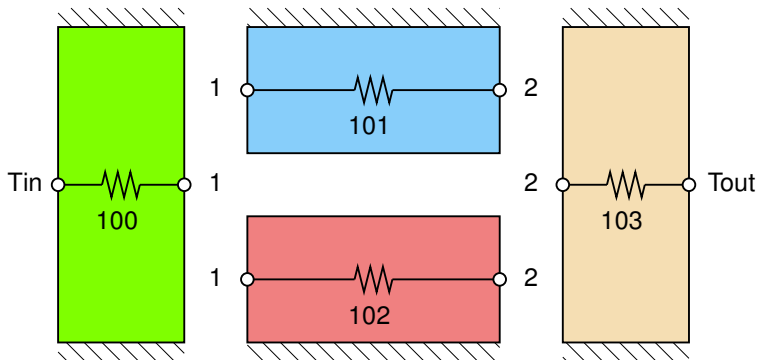
- ▶ We will examine two different thermal networks for the composite wall
- ▶ This will lead to two different TNSolver models
- ▶ Run the models for the same range of  $k_B$ :

[0.001, 0.01, 0.1, 1.0, 2.0, 3.0, 10.0, 100.0, 1000.0]

# First Approach

## Composite Wall Model 1

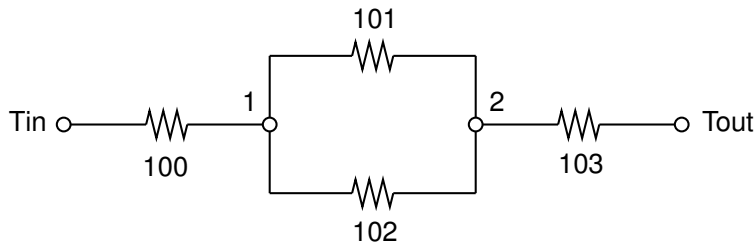
There are four control volumes:



o surface node

# Network Diagram

## Composite Wall Model 1



Compare with Figure 3.3 (a), on page 117, in [BLID11].

# TNSolver Input File for $k_B = 2.0$

## Composite Wall Model 1

Begin Solution Parameters

```
title = Composite wall model: Approach 1 series-parallel
type  = steady
```

End Solution Parameters

Begin Conductors

```
! label  type          node 1  node 2  parameters
100  conduction  Tin      1      1.0  1.0  2.0 ! k_A L_A A_A
101  conduction  1        2      2.0  3.0  1.0 ! k_B L_B A_B
102  conduction  1        2      2.0  3.0  1.0 ! k_C L_C A_C
103  conduction  2      Tout    1.0  1.0  2.0 ! k_D L_D A_D
```

End Conductors

Begin Boundary Conditions

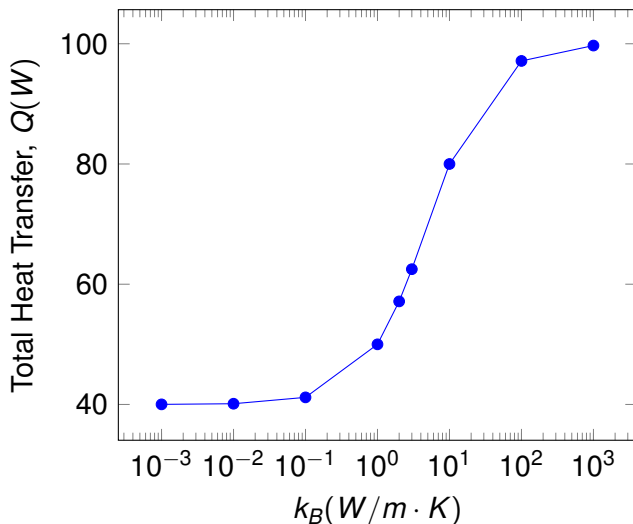
```
! type      parameter(s)  node(s)
fixed_T     100.0         Tin      ! inner wall temperature
fixed_T      0.0         Tout      ! outer wall temperature
```

End Boundary Conditions

# Total Heat Transfer over the Range of $k_B$

Composite Wall Model 1

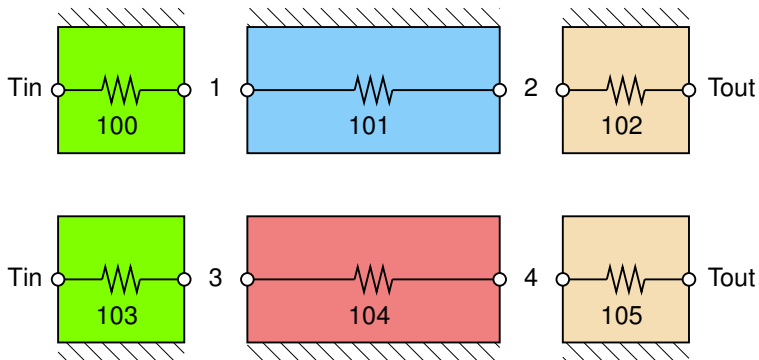
Composite Wall, First Approach



# Second Approach

## Composite Wall Model 2

There are six control volumes:

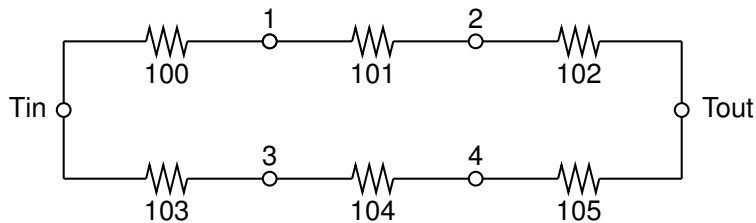


○ surface node



# Network Diagram

## Composite Wall Model 2



Compare with Figure 3.3 (b), on page 117, in [BLID11].

# TNSolver Input File for $k_B = 2.0$

## Composite Wall Model 2

Begin Solution Parameters

```
title = Composite wall model: Approach 2 - parallel conductors
type  = steady
```

End Solution Parameters

Begin Conductors

```
! label  type          node 1  node 2  parameters
100  conduction  Tin      1      1.0, 1.0, 1.0  ! k_A L_A A_B
101  conduction  1        2      2.0, 3.0, 1.0  ! k_B L_B A_B
102  conduction  2      Tout    1.0, 1.0, 1.0  ! k_D L_D A_B
103  conduction  Tin      3      1.0, 1.0, 1.0  ! k_A L_A A_C
104  conduction  3        4      2.0, 3.0, 1.0  ! k_C L_C A_C
105  conduction  4      Tout    1.0, 1.0, 1.0  ! k_D L_D A_C
```

End Conductors

Begin Boundary Conditions

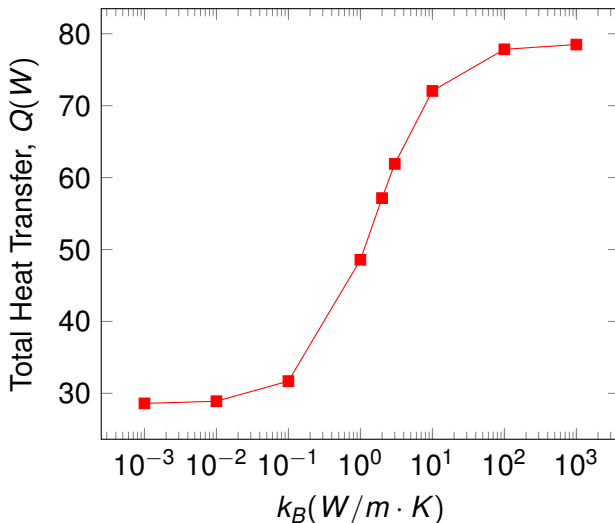
```
! type      parameter(s)  node(s)
fixed_T     100.0         Tin      ! inner wall temperature
fixed_T      0.0         Tout     ! outer wall temperature
```

End Boundary Conditions

# Total Heat Transfer over the Range of $k_B$

## Composite Wall Model 2

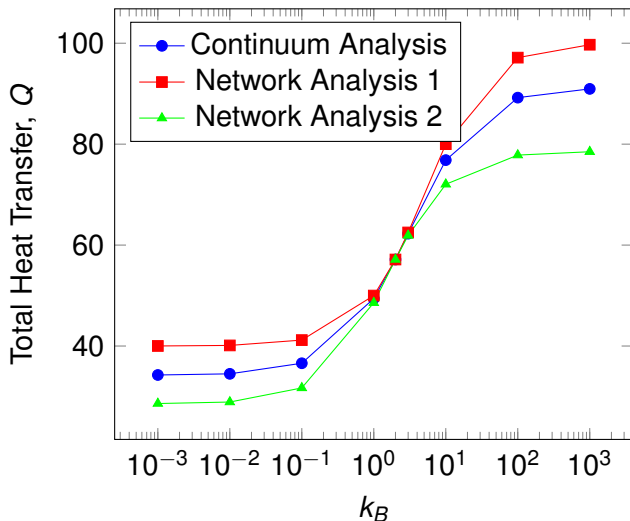
### Composite Wall, Second Approach



# Comparing Continuum to Network Analysis

## Method Comparison

### Summary of Analysis Approaches



# Conclusion

- ▶ Continuum Model
  - ▶ Solution verification for  $k_B = 2.0$
  - ▶ Grid convergence study completed
- ▶ Network Model
  - ▶ Solution verification for  $k_B = 2.0$
- ▶ The two-dimensional solution of the continuum approach is bounded by the two network models
- ▶ Network model 1 consistently over predicts the heat flow rate
- ▶ Network model 2 consistently under predicts the heat flow rate

Questions?

# References I

[BLID11] T.L. Bergman, A.S. Lavine, F.P. Incropera, and D.P. DeWitt.

*Introduction to Heat Transfer.*

John Wiley & Sons, New York, sixth edition, 2011.