Fin Convection Experiment

Thermal Network Solution with TNSolver

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Outline

- Fin Convection Experiment
- Math Model
- Thermal Network Model Analysis
- Calculations

- Four uniform cross section fins will be studied
- Steady state, forced and natural convection
- Three are circular, ●, and one is diamond shaped ◆ (a square, rotated 45°)

Fin Material	Dimensions	k (W/m-K)
Aluminum Alloy 6061-T6	0.5" ∅ X 11.22" L	167.0
Copper Alloy 110	0.5" 🗆 X 11.22" L	388.0
Copper Alloy 110	0.5" ∅ X 6.77" L	388.0
Stainless Steel Alloy	0.375" ∅ X 11.22" L	16.0

Test Setup Fin Convection Experiment



Test Fixture Assembly

Fin Convection Experiment



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Measurements

Fin Convection Experiment

The following measurements are made:

- Base temperature of the fin
- Heater power
- Ambient air temperature
- Fin temperatures along the length of the fin
 - Thermocouples are located at the center line of the fin
- Flow velocity for forced convection

A typical data set is shown for the aluminum fin, for both forced and natural convection.

Aluminum Fin - Forced Convection

Fin Convection Experiment

Heater Power = 10.53 W



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Aluminum Fin - Natural Convection

Fin Convection Experiment

Heater Power = 10.53 W



Math Model Overview

Math Model

- Conduction
- Forced Convection Correlations
- Natural or Free Convection Correlations
- Surface Radiation

Heat Conduction: Cartesian Coordinates (Plane Wall) Math Model

The rate of heat transfer, Q_{ij} , due to conduction, between the two temperatures T_i and T_j , separated by a distance *L* and area *A*, is:

$$Q_{ij}=\frac{kA}{L}\left(T_{i}-T_{j}\right)$$

The heat flux, q_{ij} , is:

$$q_{ij} = \frac{Q_{ij}}{A} = \frac{k}{L} \left(T_i - T_j \right)$$

where k is the thermal conductivity of the material.

Convection Correlations Math Model

The convection heat flow rate is:

$$Q = hA(T_s - T_\infty)$$

where *h* is the heat transfer coefficient, T_s is the surface temperature and T_{∞} is the fluid temperature

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External Forced Convection over a Cylinder

Math Model

Correlation is (Equation (7.44), page 436 in [BLID11] and [KK58]):

$$\overline{Nu}_D \equiv rac{ar{h}D}{k} = CRe_D^m Pr^{1/3}$$

where *D* is the diameter of the cylinder and the Reynolds number is $Re_D = \rho VD/\mu = VD/\nu$.

Re _D	С	т
0.4–4	0.989	0.330
4–40	0.911	0.385
40-4,000	0.683	0.466
4,000–40,000	0.193	0.618
40,000–400,000	0.027	0.805
Table 7.2, page 437 in [BLID11]		

External Forced Convection over a Noncircular Cylinder

Math Model

For the case of a gas flowing over noncircular cylinders in crossflow (Table 7.3, page 437 in [BLID11] and [SAT04]):

Geometry	Re _D	С	т
$\Rightarrow \blacklozenge D = \sqrt{2H^2}$	6,000-60,000	0.304	0.59
\Rightarrow D = H	5,000-60,000	0.158	0.66

Note that the fluid properties are evaluated at the film temperature, T_f :

$$T_f = \frac{T_s + T_\infty}{2}$$

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External Natural Convection over a Horizontal Cylinder Math Model

Correlation is (see Equation (9.34), page 581 in [BLID11] and [CC75]):

$$\overline{Nu}_{D} = \frac{\overline{h}D}{k} = \left\{ 0.60 + \frac{0.387 Ra_{D}^{1/6}}{\left[1 + (0.559/Pr)^{9/16} \right]^{8/27}} \right\}^{2}$$

valid for $Ra_D \le 10^{12}$, $Pr \ge 0.7$, where *D* is the diameter of the cylinder and the Rayleigh number, *Ra*, is:

$$Ra_{D} = GrPr = \frac{g\rho^{2}c\beta D^{3}\left(T_{s} - T_{\infty}\right)}{k\mu} = \frac{g\beta D^{3}\left(T_{s} - T_{\infty}\right)}{\nu\alpha}$$

Note that the fluid properties are evaluated at the film temperature, T_f :

$$T_f = \frac{T_s + T_\infty}{2}$$

Radiation exchange between a surface and *large* surroundings The heat flow rate is (Equation (1.7), page 10 in [BLID11]):

$$Q = \sigma \epsilon_s A_s (T_s^4 - T_{sur}^4)$$

where σ is the Stefan-Boltzmann constant, ϵ_s is the surface emissivity and A_s is the area of the surface.

Note that the surface area, A_s , must be *much* smaller than the surrounding surface area, A_{sur} :

$$A_s \ll A_{sur}$$

Note that the temperatures must be the absolute temperature, K or $^{\circ}R$

Radiation Heat Transfer Coefficient

Math Model

Define the radiation heat transfer coefficient, h_r (see Equation (1.9), page 10 in [BLID11]):

$$h_r = \epsilon \sigma (T_s + T_{sur}) (T_s^2 + T_{sur}^2)$$

Then,

$$Q = h_r A_s (T_s - T_{sur})$$

Note:

- h_r is temperature dependent
- h_r can be used to compare the radiation to the convection heat transfer from a surface, h (if T_{sur} and T_{∞} have similar values)

Thermocouple Locations for Aluminum Fin

Thermal Network Model Analysis



Position Along the Fin (m)						
Base	TC1	TC2	TC3	TC4	TC5	TC6
0.0	.03	.055	.1	.170	.220	.283

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Long Fin Thermal Network Model

Thermal Network Model Analysis





Short Copper Fin Thermal Network Model

Thermal Network Model Analysis

For the short copper fin, there are four thermocouples:



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TNSolver Input Files

Thermal Network Model Analysis

The supplied TNSolver input files for each fin model:

Fin	Forced Convection	Natural Convection
Aluminum, 11.22"	Al_fin_FC.inp	Al_fin_NC.inp
Round Copper, 6.77"	Cu_fin_FC.inp	Cu_fin_NC.inp
Square Copper, 11.22"	sqCu_fin_FC.inp	sqCu_fin_NC.inp
Stainless Steel, 11.22"	SS_fin_FC.inp	SS_fin_NC.inp

Edit each file to set appropriate boundary conditions (Tc, Tr and base) and flow velocities (forced convection models) for your specific experimental data set

TNSolver Input Files for Estimating h

Thermal Network Model Analysis

The supplied TNSolver input files for each fin model to be used with ls_fin_h.m

Fin	Convection
Aluminum, 11.22"	Al_fin_C.inp
Round Copper, 6.77"	Cu_fin_C.inp
Square Copper, 11.22"	sqCu_fin_C.inp
Stainless Steel, 11.22"	SS_fin_C.inp

Edit each file to set appropriate boundary conditions (Tc, Tr and base) for your specific experimental data set

Running a TNSolver Model

Thermal Network Model Analysis

The MATLAB/Octave command to run the aluminum fin, forced convection model is:

```
>>tnsolver('Al_fin_FC');
```

Note that you supply the base of the input file name (AL_fin_FC, in this example), without the .inp extension

The results output file will be the base name with the extension .out (Al_fin_FC.out, in this example)

Surface Radiation Properties

Thermal Network Model Analysis

	Emissivity, ϵ	
Fin Material	(dimensionless)	
Aluminum Alloy 6061-T6	0.09	
Copper Alloy 110	0.12-0.15	
Stainless Steel Alloy	0.5-0.7	
See Table A.8, page 922, in [BLID11]		
See Table 10.1, page 530, in [LL16]		

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Verification with Analytical Solution

Thermal Network Model Analysis

Table 3.4, page 161 in [BLID11], Temperature distribution and heat loss for fins with a uniform cross section

Case	Tip Condition	Temperature, θ/θ_b	Fin Heat Transfer Rate, Q
Α	Convection	$\frac{\cosh m(L-x) + (h/mk) \sinh m(L-x)}{\cosh mL + (h/mk) \sinh ml}$	$M_{\overline{\cosh mL+(h/mk)} \cosh mL}^{\sinh mL+(h/mk) \cosh mL}$
В	Adiabatic	$\frac{\cosh m(L-x)}{\cosh mL}$	<i>M</i> tanh <i>mL</i>
С	Specified T	$\frac{(\theta_L/\theta_b)\sinh mx + \sinh m(L-x)}{\sinh ml}$	$M rac{\cosh mL - (heta_L/ heta_b)}{\sinh mL}$
D	$L = \infty$	e ^{mx}	M
$\theta = T$	$-T_{\infty}$	$m = \sqrt{hP/kA_c}$	
$\theta_b = \theta$	$(0)=T_b-T_\infty$	$M = (\sqrt{hPkA_c})\theta_b$	

For circular fins: $P = \pi D$ and $A_c = \pi (D/2)^2$ For square/diamond fins: P = 4H and $A_c = H^2$

Comparison with TNSolver Network Model

Thermal Network Model Analysis



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Calculations

Calculations

For the four forced convection experimental data sets:

- 1. Using forced convection correlations, determine h and h_r for each fin
 - EFCcyl and EFCdiamond conductors
- 2. Using least-squares fit, estimate ϵ , using forced convection correlations
 - ls_fin_emiss.m function
- 3. Using least-squares fit, estimate h, using estimated ϵ and forced convection correlations
 - ls_fin_h.m function

How does the estimated h compare to the correlation?

How does the estimated ϵ compare to the material property table value?

Calculations (continued)

Calculations

For four free convection experimental data sets:

- 1. Using free convection correlations, determine h and h_r for each fin
 - ENChcyl conductor
- 2. Using least-squares fit, estimate ϵ , using free convection correlation
 - ls_fin_emiss.m function
- 3. Using least-squares fit, estimate h, using estimated ϵ and free convection correlation
 - ls_fin_h.m function

How does the estimated h compare to the correlation?

How does the estimated ϵ compare to the material property table value?

Least Squares Estimation

Calculations

Least squares estimation of a parameter θ , for *n* data points, is:

$$R = \sum_{i=1}^{n} \left(heta_{\mathsf{exp}}^{i} - heta_{\mathsf{model}}^{i}
ight)^{2}$$

The closest model parameter is determined by the index i, of the minimum R

Function to estimate emissivity, ϵ , is: ls_fin_emiss.m

Function to estimate convection coefficent, *h*, is: ls_fin_h.m

Conclusion

- Fin convection experiment overview
- Conduction, convection and surface radiation math model
- Thermal network model analysis
- Calculations using the experimental data

Questions?

Appendix

Heat Transfer Analysis

Thermal Network Model

- Energy conservation: control volumes
- Identify and sketch out the control volumes
- Use the conductor analogy to represent energy transfer between the control volumes and energy generation or storage
 - Conduction, convection, radiation, other?
 - Capacitance
 - Sources or sinks
- State assumptions and determine appropriate parameters for each conductor
 - Geometry, material properties, etc.
- Which conductor(s)/source(s)/capacitance(s) are important to the required results?
 - Sensitivity analysis
- What is missing from the model? peer/expert review

Thermal Network Terminology

Thermal Network Model

- Geometry
 - Control Volume
 - Volume property, $V = \int_V dV$
 - Node: •, $T_{node} = \int_V T(x_i) dV$
 - Control Volume Surface
 - Area property, $A = \int_A dA$
 - Surface Node: \bigcirc , $T_{\text{surface node}}^{n} = \int_{A} T(x_i) dA$
- Material properties
- Conductors
 - Conduction, convection, radiation
- Boundary conditions
 - Boundary node: A

Thermal Network Analysis

Thermal Network Model

For the 11.22" long fin, there are six thermocouples:



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Thermal Network Analysis

Thermal Network Model

For the 6.77" long fin, there are four thermocouples:



Control Volume Geometry Thermal Network Model

Scale drawing of thermocouple locations and control volume lengths:



Note that the control volume boundaries are located midway between the thermocouple locations

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Round Fin, 11.22" Long, Conduction Geometry Thermal Network Model

Axial conduction in the fin, $L = 11.22^{\circ} = 0.285$ m D = 0.5" = 0.0127 m and r = 0.25" = 0.00635 m:

L = distance between TC

Conductor A (m^2) Length (*m*) 0.030 0.00012667 100 101 0.025 0.00012667 102 0.045 0.00012667 103 0.070 0.00012667 104 0.050 0.00012667 105 0.063 0.00012667 106 0.00012667 0.002

 $A = \pi r^2$

Round Fin, 11.22" Long, Convection Geometry

Thermal Network Model

Surface convection area:

$$A = L_{CV}P = L_{CV}\pi D$$

D = 0.5" = 0.0127 m

Conductor	L_{CV} (m)	A (m ²)
121	0.0425	0.001696
122	0.0350	0.001396
123	0.0575	0.002294
124	0.0600	0.002394
125	0.0565	0.002254
126	0.0335	0.001337

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Round Fin, 6.77" Long, Conduction Geometry Thermal Network Model

Axial conduction in the fin, L = 6.77" = 0.172 mD = 0.5" = 0.0127 m and r = 0.25" = 0.00635 m:

Conductor	Length (m)	A (<i>m</i> ²)
100	0.030	0.00012667
101	0.025	0.00012667
102	0.045	0.00012667
103	0.070	0.00012667
104	0.002	0.00012667

L = distance between TC $A = \pi r^2$

Round Fin, 6.77" Long, Convection Geometry

Thermal Network Model

Surface convection area:

$$A = L_{CV}P = L_{CV}\pi D$$

D = 0.5" = 0.0127 m

$L_{CV}(m)$	A (m ²)
0.0425	0.001696
0.0350	0.001396
0.0575	0.002294
0.0370	0.001476
	L _{CV} (m) 0.0425 0.0350 0.0575 0.0370

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Square Fin, 11.22" Long, Conduction Geometry Thermal Network Model

Axial conduction in the fin, L = 11.22" = 0.285 m, H = 0.5" = 0.0127 m

L = distance between TC $A = H^2$

Conductor	Length (m)	A (<i>m</i> ²)
100	0.030	0.000016129
101	0.025	0.000016129
102	0.045	0.000016129
103	0.070	0.000016129
104	0.050	0.000016129
105	0.063	0.000016129
106	0.002	0.000016129

Square Fin, 11.22" Long, Convection Geometry

Thermal Network Model

Surface convection area:

$$A = L_{CV}P = L_{CV}4H$$

H = 0.5" = 0.0127 m

Conductor	$L_{CV}(m)$	A (m ²)
121	0.0425	0.002159
122	0.0350	0.001778
123	0.0575	0.002921
124	0.0600	0.003048
125	0.0565	0.0028702
126	0.0335	0.0017018

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Conduction Conductor

Thermal Network Model

$$Q_{ij}=\frac{kA}{L}(T_i-T_j)$$

labe	l type	nd_i n	d_j	k	L	A
100	conduction	base	1	х.х	0.030	0.00012667
101	conduction	1	2	х.х	0.025	0.00012667
102	conduction	2	3	х.х	0.045	0.00012667
103	conduction	3	4	х.х	0.070	0.00012667
104	conduction	4	5	х.х	0.050	0.00012667
105	conduction	5	6	х.х	0.063	0.00012667
106	conduction	6	tip	х.х	0.002	0.00012667

Convection Conductor

Thermal Network Model

$$Q_{ij} = hA(T_s - T_\infty) = hA(T_i - T_j)$$

```
Begin Conductors
! label type nd_i nd_j h A
    121 convection 1 Tc x.x 0.001696
    122 convection 2 Tc x.x 0.001396
    123 convection 3 Tc x.x 0.002294
    124 convection 4 Tc x.x 0.002394
    125 convection 5 Tc x.x 0.002254
    126 convection 6 Tc x.x 0.001337
    127 convection tip Tc x.x 0.00012667
End Conductors
```

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Forced Convection Over Cylinder Conductor

Thermal Network Model

Heat transfer coefficient, h, is evaluated using the correlation.

```
      Begin Conductors

      ! label type nd_i nd_j fluid V D A

      121 EFCcyl 1 Tc air x.x 0.0127 0.001696

      122 EFCcyl 2 Tc air x.x 0.0127 0.001396

      123 EFCcyl 3 Tc air x.x 0.0127 0.002294

      124 EFCcyl 4 Tc air x.x 0.0127 0.002394

      125 EFCcyl 5 Tc air x.x 0.0127 0.002254

      126 EFCcyl 6 Tc air x.x 0.0127 0.001337

      127 EFCcyl tip Tc air x.x 0.0127 0.00012667

      End Conductors
```

Note that Re, Nu and h are reported in the output file.

Forced Convection Over Diamond Conductor

Thermal Network Model

Heat transfer coefficient, h, is evaluated using the correlation.

```
Begin Conductors

! label type nd_i nd_j fluid V D A

121 EFCdiamond 1 Tc air x.x 0.018 0.002159

122 EFCdiamond 2 Tc air x.x 0.018 0.001778

123 EFCdiamond 3 Tc air x.x 0.018 0.002921

124 EFCdiamond 4 Tc air x.x 0.018 0.003048

125 EFCdiamond 5 Tc air x.x 0.018 0.0028702

126 EFCdiamond 6 Tc air x.x 0.018 0.0017018

127 EFCdiamond tip Tc air x.x 0.0127 0.00012667

End Conductors
```

Note that Re, Nu and h are reported in the output file.

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Natural Convection Conductor

Thermal Network Model

Heat transfer coefficient, h, is evaluated using the correlation.

```
      Begin Conductors

      ! label type nd_i nd_j fluid D
      A

      121 ENChcyl 1
      Tc air 0.0127
      0.001696

      122 ENChcyl 2
      Tc air 0.0127
      0.001396

      123 ENChcyl 3
      Tc air 0.0127
      0.002294

      124 ENChcyl 4
      Tc air 0.0127
      0.002394

      125 ENChcyl 5
      Tc air 0.0127
      0.002254

      126 ENChcyl 6
      Tc air 0.0127
      0.001337

      127 ENChcyl tip
      Tc air 0.0127
      0.00012667
```

Note that Ra, Nu and h are reported in the output file.

Surface Radiation Conductor

Thermal Network Model

 $Q_{ij} = \sigma \epsilon_i A_i (T_i^4 - T_j^4)$

! labe	l type	nd_i	nd_j	emissivity	A
221	surfrad	1	Tr	x.xx -	0.001696
222	surfrad	2	Tr	X.XX	0.001396
223	surfrad	3	Tr	X.XX	0.002294
224	surfrad	4	Tr	X.XX	0.002394
225	surfrad	5	Tr	X.XX	0.002254
226	surfrad	6	Tr	X.XX	0.001337
227	surfrad	tip	Tr	X.XX	0.00012667

Note that h_r is reported in the output file.

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Boundary Conditions

Thermal Network Model

```
Begin Boundary Conditions

! type Tb Node(s)

fixed_T x.xx Tc ! fluid T

fixed_T x.xx Tr ! surrounding radiation T

fixed_T x.xx base ! fin base T

End Boundary Conditions
```

The supplied TNSolver input files for each fin model:

Fin	Forced Convection	Natural Convection
Aluminum, 11.22"	Al_fin_FC.inp	Al_fin_NC.inp
Round Copper, 6.77"	Cu_fin_FC.inp	Cu_fin_NC.inp
Square Copper, 11.22"	sqCu_fin_FC.inp	sqCu_fin_NC.inp
Stainless Steel, 11.22"	SS_fin_FC.inp	SS_fin_NC.inp

Edit each file to set appropriate boundary conditions (Tc, Tr and base) and flow velocities (forced convection models) for your specific experimental data set

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Running a TNSolver Model

Thermal Network Model

The MATLAB/Octave command to run the aluminum fin, forced convection model is:

```
>>tnsolver('Al_fin_FC');
```

Note that you supply the base of the input file name (AL_fin_FC, in this example), without the .inp extension

The results output file will be the base name with the extension .out (Al_fin_FC.out, in this example)

Screen Output When Running TNSolver

Thermal Network Model

```
*****
       TNSolver - A Thermal Network Solver
         Version 0.2.0, October 28, 2014
Reading the input file: Al_fin_FC.inp
Initializing the thermal network model ...
Starting solution of a steady thermal network model ...
   Nonlinear Solve
 Iteration Residual
 _____
        52.9978
0.0309635
     1
     2
     3
        1.21353e-05
         5.09316e-09
     4
Results have been written to: Al_fin_FC.out
All done ...
```

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View TNSolver Output File For Results

Thermal Network Model

Open Al_fin_FC.out in your favorite text editor

View TNSolver Output File For Results (continued)

Thermal Network Model

		Volume	Temperature
Label	Material	(m^3)	(C)
1	N/A	0	47.1038
2	N/A	0	42.202
3	N/A	0	36.0832
4	N/A	0	31.0199
5	N/A	0	29.207
6	N/A	0	28.4181
Тс	N/A	0	25
Tr	N/A	0	25
base	N/A	0	55.8
tip	N/A	0	28.416

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View TNSolver Output File For Results (continued)

Thermal Network Model

əl	Туре	Node i	Node j	Q_ij (W)
100	conduction	base	1	6.13192
101	conduction	1	2	4.14771
102	conduction	2	3	2.87638
103	conduction	3	4	1.53011
104	conduction	4	5	0.766993
105	conduction	5	6	0.264889
106	conduction	6	tip	0.0229109
121	EFCcyl	1	Tc	1.96157
122	EFCcyl	2	Tc	1.25717
123	EFCcyl	3	Tc	1.33172
124	EFCcyl	4	Tc	0.755085
125	EFCcyl	5	Tc	0.496864
126	EFCcyl	6	Tc	0.239463
127	EFCcyl	tip	Tc	0.0226728
221	surfrad	1	Tr	0.0226511
222	surfrad	2	Tr	0.0141605
223	surfrad	3	Tr	0.0145417
224	surfrad	4	Tr	0.00803642
225	surfrad	5	Tr	0.00523996
226	surfrad	6	Tr	0.00251536
227	surfrad	tip	Tr	0.000238156

View TNSolver Output File For Results (continued)

Thermal Network Model

*** Conductor Parameters *** surfrad: Surface Radiation h_r label (W/m^2-K) _____ ____ 221 0.60422 222 0.589676 223 0.571949 0.557634 224 225 0.552586 226 0.550402 227 0.550396 EFCcyl: External Forced Convection - Cylinder h label Re Number Nu Number (W/m^2-K) 121 2832.69 24.7609 52.325 1222869.3624.919252.35151232917.0225.120752.37891242958.5325.290952.39411252973.9425.352652.3974 126 2980.75 25.3797 52.3984 127 2980.77 25.3798 52.3984

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